

19/9/2023

DISEQUAZIONI IRRAZIONALI

$$\sqrt{f(x)} < g(x)$$

$$\begin{cases} f(x) \geq 0 \\ g(x) > 0 \\ f(x) < g^2(x) \end{cases}$$

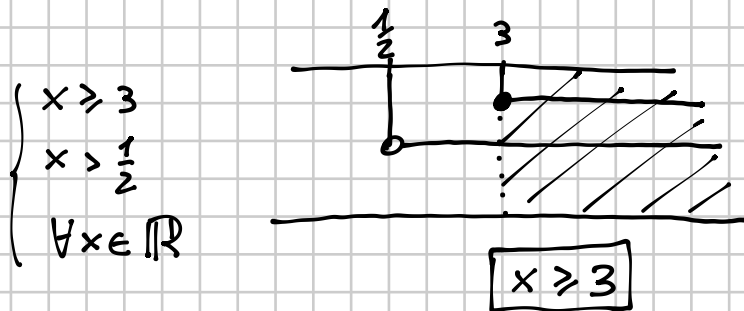
768

$$\sqrt{x-3} < 2x-1$$

$$[x \geq 3]$$

$$\begin{cases} x-3 \geq 0 \\ 2x-1 > 0 \\ x-3 < (2x-1)^2 \end{cases} \quad \begin{cases} x \geq 3 \\ x > \frac{1}{2} \\ x-3 < 4x^2+1-4x \\ 4x^2-5x+4 > 0 \end{cases} \quad \begin{cases} x \geq 3 \\ x > \frac{1}{2} \\ 4x^2-5x+4 > 0 \end{cases}$$

$$\Delta = 25 - 48 < 0 \quad \forall x \in \mathbb{R}$$



776

$$2\sqrt{x^2 - 5x + 7} \leq 2x - 4$$

$$[x \geq 3]$$

$$\begin{cases} x^2 - 5x + 7 \geq 0 \\ 2x - 4 \geq 0 \\ \cancel{f(x^2 - 5x + 7)} \leq \cancel{f(x-2)^2} \end{cases} \begin{matrix} \leftarrow \Delta < 0 \\ \underbrace{\hspace{10em}}_{2(x-2)} \end{matrix} \quad \begin{cases} \forall x \in \mathbb{R} \\ x \geq 2 \\ \cancel{x^2 - 5x + 7} \leq \cancel{x^2 + 4 - 4x} \end{cases}$$

$$\begin{cases} \forall x \in \mathbb{R} \\ x \geq 2 \\ -x \leq -3 \end{cases} \quad \begin{cases} \forall x \in \mathbb{R} \\ x \geq 2 \\ x \geq 3 \end{cases} \Rightarrow \boxed{x \geq 3}$$

$$\sqrt{f(x)} > g(x)$$

$$\begin{cases} g(x) < 0 \\ f(x) \geq 0 \end{cases} \vee \begin{cases} g(x) \geq 0 \\ f(x) > g^2(x) \end{cases}$$

792

$$\sqrt{x(x-4)+4} > 2x+1$$

$$\left[x < \frac{1}{3} \right]$$

$$\begin{cases} 2x+1 < 0 \\ x(x-4)+4 \geq 0 \end{cases} \vee \begin{cases} 2x+1 \geq 0 \\ x(x-4)+4 > (2x+1)^2 \end{cases}$$

$$\begin{cases} x < -\frac{1}{2} \\ x^2 - 4x + 4 \geq 0 \\ (x-2)^2 \geq 0 \end{cases} \vee \begin{cases} x \geq -\frac{1}{2} \\ x^2 - 4x + 4 > 4x^2 + 4x + 1 \\ -3x^2 - 8x + 3 > 0 \end{cases}$$

$$\begin{cases} x < -\frac{1}{2} \\ \forall x \in \mathbb{R} \end{cases} \Downarrow x < -\frac{1}{2}$$

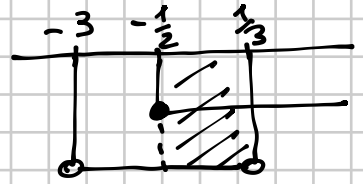
$$3x^2 + 8x - 3 < 0 \quad \Delta = 16 + 9 = 25$$

$$-3 < x < \frac{1}{3} \quad x = \frac{-4 \pm 5}{3} = \begin{cases} -3 \\ \frac{1}{3} \end{cases}$$

$$x < -\frac{1}{2}$$

v

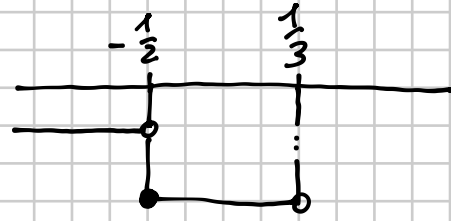
$$\begin{cases} x \geq -\frac{1}{2} \\ -3 < x < \frac{1}{3} \end{cases}$$



⇓

$$-\frac{1}{2} \leq x < \frac{1}{3}$$

$$x < -\frac{1}{2} \quad \vee \quad -\frac{1}{2} \leq x < \frac{1}{3}$$



$$x < \frac{1}{3}$$

742

$$\sqrt{3x+13} - \sqrt{3(x+2)} = \sqrt{x+3} - \sqrt{x}$$

[1]

$$\sqrt{3x+13} + \sqrt{x} = \sqrt{x+3} + \sqrt{3(x+2)}$$

$$\begin{cases} 3x+13 \geq 0 \\ x \geq 0 \\ x+3 \geq 0 \\ x+2 \geq 0 \end{cases} \Rightarrow \begin{cases} x \geq -\frac{13}{3} \\ x \geq 0 \\ x \geq -3 \\ x \geq -2 \end{cases} \Rightarrow x \geq 0$$

$$\begin{cases} x \geq 0 \end{cases}$$

$$\cancel{3x+13} + \cancel{x} + 2\sqrt{(3x+13)x} = \cancel{x+3} + \cancel{3x+6} + 2\sqrt{3(x+2)(x+3)}$$

$$\begin{cases} x \geq 0 \end{cases}$$

$$\cancel{4} + \cancel{2}\sqrt{x(3x+13)} = \cancel{2}\sqrt{3(x+2)(x+3)}$$

$$\begin{cases} x \geq 0 \end{cases}$$

$$2 + \sqrt{x(3x+13)} = \sqrt{3(x+2)(x+3)}$$

$$\begin{cases} x \geq 0 \\ 4 + x(3x+13) + 4\sqrt{x(3x+13)} = 3(x+2)(x+3) \end{cases}$$

$$\begin{cases} x \geq 0 \\ 4\sqrt{3x^2+13x} = 3(x^2+3x+2x+6) - 4 - 3x^2 - 13x \end{cases}$$

$$\begin{cases} x \geq 0 \\ 4\sqrt{3x^2+13x} = \cancel{3x^2} + 15x + 18 - 4 - \cancel{3x^2} - 13x \end{cases}$$

$$\begin{cases} x \geq 0 \\ 4\sqrt{3x^2+13x} = 2x + 14 \end{cases} \quad \begin{cases} x \geq 0 \\ 2\sqrt{3x^2+13x} = x + 7 \end{cases}$$

$$\begin{cases} x \geq 0 \\ x+7 \geq 0 \end{cases} \quad \begin{cases} x \geq 0 \\ x \geq -7 \end{cases} \Rightarrow x \geq 0$$

$$\begin{cases} 4(3x^2+13x) = x^2 + 49 + 14x \\ 12x^2 + 52x = x^2 + 49 + 14x \end{cases}$$

$$\begin{cases} x \geq 0 \\ 11x^2 + 38x - 49 = 0 \end{cases} \quad \Delta = 19^2 + 11 \cdot 49 = 361 + 539 = 900 = 30^2$$

$$x = \frac{-19 \pm 30}{11} = \begin{cases} -\frac{49}{11} \text{ N.A.C.C. perché } x \geq 0 \\ \frac{11}{11} = 1 \end{cases} \quad \boxed{x=1}$$