615 Studia il fascio di rette di equazione $(2k+1)x+(3+k)y+1-2k=0$, determinando le equazioni delle generatrici e le coordinate del centro C. Calcola il valore di k corrispondente alla retta:

a. parallela alla retta di equazione $x+y-1=0$;

b. passante per $P(5;1)$;

c. passante per Q, essendo Q un punto del primo quadrante, vertice del triangolo isoscele PCQ di base PC e area $\frac{441}{40}$.

$\left[\text{generatrici:} x+3y+1=0,\ 2x+y-2=0;\ C\left(\frac{7}{5};-\frac{4}{5}\right);\ \text{a)}2;\ \text{b)}-1;\ \text{c)}-\frac{67}{18}\right]$

$$2kx+x+3y+ky+1-2k=0$$

$$x+3y+1+k(2x+y-2)=0$$

1ª GEN.: $x+3y+1=0$

2ª GEN.: $2x+y-2=0$ (retta esclusa)

$$C\begin{cases}x+3y+1=0\\2x+y-2=0\end{cases}\qquad\begin{cases}x=-1-3y\\-2-6y+y-2=0\end{cases}$$

$$\begin{cases}\\-5y=4\end{cases}\qquad\begin{cases}x=-1+\frac{12}{5}=\frac{7}{5}\\y=-\frac{4}{5}\end{cases}\qquad C\left(\frac{7}{5},-\frac{4}{5}\right)$$

a) $x+y-1=0\qquad y=-x+1\qquad m=-1$

$$-\frac{2k+1}{3+k}=-1\qquad 2k+1=3+k\Rightarrow\boxed{k=2}$$

$$\Downarrow$$

$$5x+5y-3=0$$

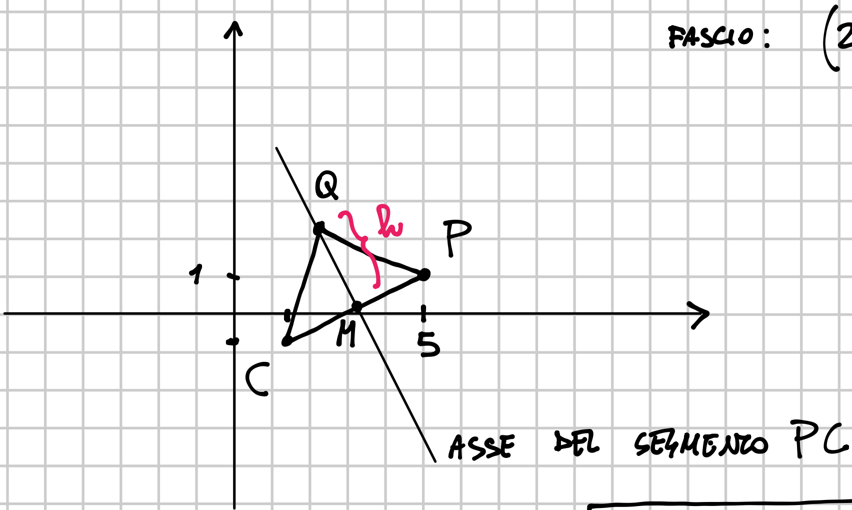
b) $(2k+1)x+(3+k)y+1-2k=0\qquad P(5,1)$

$$(2k+1)\cdot5+(3+k)\cdot1+1-2k=0$$

$$10k+5+3+k+1-2k=0$$

$$9k=-9\Rightarrow\boxed{k=-1}$$

c)



FASCIO: $(2K+1)x+(3+K)y+1-2K=0$

$$\mathcal{A}_{PCQ}=\frac{441}{40}$$

$P(5,1)$ $C\left(\frac{7}{5},-\frac{4}{5}\right)$

$$\overline{PC}=\sqrt{\left(5-\frac{7}{5}\right)^2+\left(1+\frac{4}{5}\right)^2}=\sqrt{\left(\frac{18}{5}\right)^2+\left(\frac{9}{5}\right)^2}=$$

$$=\frac{1}{5}\sqrt{4\cdot 9^2+9^2}=\frac{1}{5}\sqrt{5\cdot 9^2}=\frac{9}{5}\sqrt{5}$$

$$M\left(\frac{5+\frac{7}{5}}{2},\frac{1-\frac{4}{5}}{2}\right)=$$

$$=\left(\frac{16}{5},\frac{1}{10}\right)$$

$$h=\frac{2\mathcal{A}}{\overline{PC}}=\frac{\frac{441}{20}}{\frac{9}{5}\sqrt{5}}=\frac{\cancel{441}^{49}}{\cancel{20}_{4}}\cdot\frac{\cancel{5}^{1}}{\cancel{9}\sqrt{5}}=\frac{49}{4\sqrt{5}}$$

$$m_{PC}=\frac{1+\frac{4}{5}}{5-\frac{7}{5}}=\frac{\frac{9}{\cancel{5}}}{\frac{18}{\cancel{5}}}=\frac{1}{2}$$

ASSE SEGMENTO PC

$$y-\frac{1}{10}=-2\left(x-\frac{16}{5}\right)$$

$$y=-2x+\frac{32}{5}+\frac{1}{10}$$

$$y=-2x+\frac{\cancel{65}^{13}}{\cancel{10}_{2}}\qquad y=-2x+\frac{13}{2}$$

$Q\left(x,-2x+\frac{13}{2}\right)$ ← CONDIZIONE DI APPARTENENZA DI Q ALLA RETTA

Vado a cercare i due punti Q che distano da $M\left(\frac{16}{5},\frac{1}{10}\right)$ esattamente $h=\frac{49}{4\sqrt{5}}$

$$\overline{QM}=\frac{49}{4\sqrt{5}}\iff\overline{QM}^2=\left(\frac{49}{4\sqrt{5}}\right)^2$$

$$\left(x-\frac{16}{5}\right)^2+\left(-2x+\frac{13}{2}-\frac{1}{10}\right)^2=\left(\frac{49}{4\sqrt{5}}\right)^2$$

eq. 2° grado
risolta con WOLFRAM ALPHA

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$$x=\frac{3}{4}\quad\vee\quad x=\frac{113}{20}$$

$Q\left(x, -2x + \frac{13}{2}\right)$

$x = \frac{3}{4} \Rightarrow y = -2 \cdot \frac{3}{4} + \frac{13}{2} = -\frac{3}{2} + \frac{13}{2} = \frac{10}{2} = 5 \qquad Q\left(\frac{3}{4}, 5\right)$

$x = \frac{113}{20} \Rightarrow y = -\cancel{2} \cdot \frac{113}{\cancel{20}_{10}} + \frac{13}{2} = \frac{-113 + 65}{10} < 0$ NON ACC. perché $y_Q > 0$

Trovo la retta del fascio per Q

$(2K+1)x + (3+K)y + 1 - 2K = 0$

$(2K+1) \cdot \frac{3}{4} + (3+K) \cdot 5 + 1 - 2K = 0$

$\Big\downarrow$ CON WOLFRAM ALPHA

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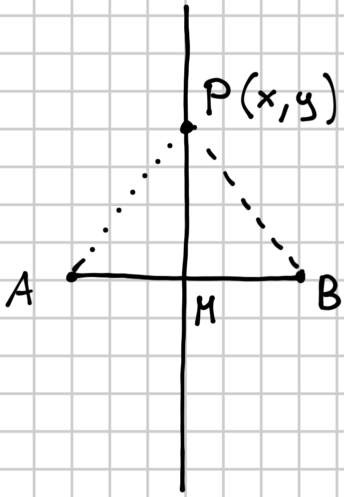
$\boxed{K = -\frac{67}{18}}$

ASSE DI UN SEGMENTO = LUOGO GEOMETRICO dei punti che hanno la stessa distanza dagli estremi

$A(x_A, y_A) \qquad B(x_B, y_B)$



$$\Rightarrow \quad \overline{PA} = \overline{PB}$$

$$\sqrt{(x-x_A)^2+(y-y_A)^2} = \sqrt{(x-x_B)^2+(y-y_B)^2}$$

$$(x-x_A)^2+(y-y_A)^2 = (x-x_B)^2+(y-y_B)^2$$

ESEMPIO

$A\left(\frac{7}{5}, -\frac{4}{5}\right) \qquad B(5,1)$

$$\left(x-\frac{7}{5}\right)^2+\left(y+\frac{4}{5}\right)^2 = (x-5)^2+(y-1)^2$$

$$\cancel{x^2}+\frac{49}{25}-\frac{14}{5}x+\cancel{y^2}+\frac{16}{25}+\frac{8}{5}y = \cancel{x^2}+25-10x+\cancel{y^2}+1-2y$$

$$\frac{8}{5}y+2y = \frac{14}{5}x-10x-\frac{49}{25}-\frac{16}{25}+25+1$$

$$\frac{18}{5}y = -\frac{36}{5}x-\frac{\cancel{65}^{\,13}}{\cancel{25}_{\,5}}+26$$

$$\frac{18}{\cancel{5}}y = -\frac{36}{\cancel{5}}x+\frac{117}{\cancel{5}}$$

$$2y = -4x+13$$

$$y = -2x+\frac{13}{2}$$