242 $4^{\sqrt{x+2}} + 6 = 4^{2-\sqrt{x+2}}$ $\left[-\frac{7}{4}\right]$

C.E.

$x+2 \geq 0 \qquad x \geq -2$

$4^{\sqrt{x+2}} + 6 = \frac{4^2}{4^{\sqrt{x+2}}}$

$4^{\sqrt{x+2}} = t$

$t + 6 = \frac{16}{t} \qquad t \neq 0$

$t^2 + 6t = 16$

$t^2 + 6t - 16 = 0 \qquad \frac{\Delta}{4} = 9 + 16 = 25$

$t = -3 \pm 5 = \begin{cases} -8 \Rightarrow 4^{\sqrt{x+2}} = -8 \text{ IMPOSS.} \\ 2 \end{cases}$

$4^{\sqrt{x+2}} = 2$

$2^{2\sqrt{x+2}} = 2$

$\Downarrow$

$2\sqrt{x+2} = 1$

elevo al quadrato

$4(x+2) = 1$

$x + 2 = \frac{1}{4} \qquad x = \frac{1}{4} - 2 = -\frac{7}{4}$

250 $\left(\frac{2}{5}\right)^{x-1} - \left(\frac{5}{2}\right)^{\frac{x-1}{x}} = 0$ [±1]

C.E.
$x \neq 0$

$$\left(\frac{2}{5}\right)^{x-1} - \left(\frac{2}{5}\right)^{-\frac{x-1}{x}} = 0$$

$$\left(\frac{2}{5}\right)^{x-1} = \left(\frac{2}{5}\right)^{-\frac{x-1}{x}}$$

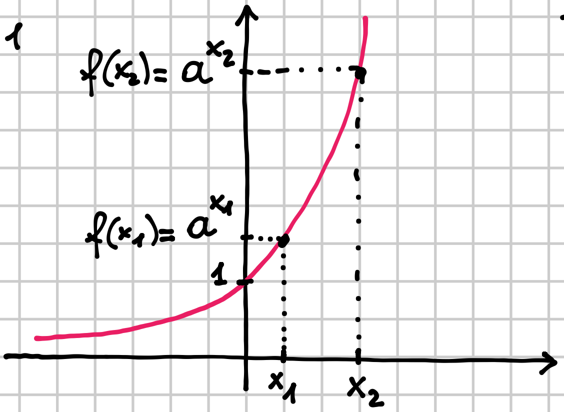
$$x-1 = -\frac{x-1}{x}$$

$$x^2 \cancel{-x} = \cancel{-x} + 1$$

$$x^2 = 1 \qquad \boxed{x = \pm 1}$$

DISEQUAZIONI ESPONENZIALI

$f(x) = a^x \qquad a > 1$



f è STRETTAMENTE CRESCENTE

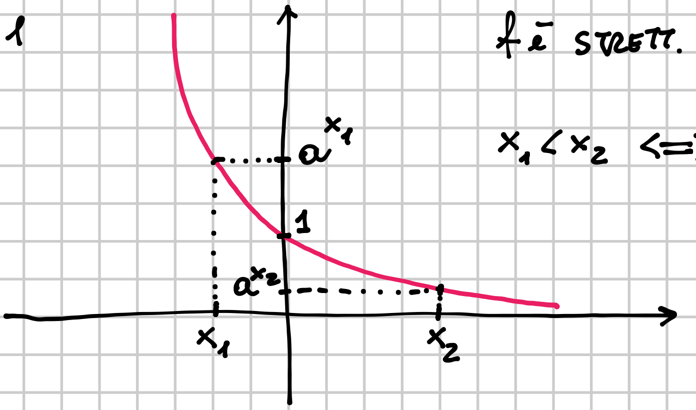
$x_1 < x_2 \Leftrightarrow f(x_1) < f(x_2)$

cioè

$a^{x_1} < a^{x_2}$

Se ho una disequazione del tipo $2^x < 2^5$ posso passare agli esponenti mantenendo la stessa disuguaglianza, cioè $x < 5$ (perché la base 2 è maggiore di 1)

$f(x) = a^x \qquad 0 < a < 1$



f è STRETT. DECRESCENTE

$x_1 < x_2 \Leftrightarrow f(x_1) > f(x_2)$

$a^{x_1} > a^{x_2}$

Se ho un'eq. del tipo $\left(\frac{1}{2}\right)^x > \left(\frac{1}{2}\right)^3$ passo a $x < 3$

devo invertire la disuguaglianza perché $\frac{1}{2} < 1$

269 $\left(\frac{3}{2}\right)^x < \frac{8}{27}$

$\left(\frac{3}{2}\right)^x < \left(\frac{2}{3}\right)^3$

ALTERNATIVA

$\left(\frac{2}{3}\right)^{-x} < \left(\frac{2}{3}\right)^3$ inverto la disug. perché $\frac{2}{3} < 1$

$-x > 3$

$x < -3$

$\left(\frac{3}{2}\right)^x < \left(\frac{3}{2}\right)^{-3}$

$x < -3$ perché $\frac{3}{2} > 1$

277 $5^{x^2-1} > \left(\frac{1}{5}\right)^{3x+1}$ $[x < -3 \vee x > 0]$

$5^{x^2-1} > 5^{-3x-1}$

$x^2 \cancel{-1} > -3x \cancel{-1}$

$x^2 + 3x > 0$

$x(x+3) > 0$

$x < -3 \vee x > 0$

280 $17 \cdot \sqrt{2^{x+1}} > 34 \cdot \sqrt[3]{4^{x-3}}$ $[x < 9]$

$$\cancel{17}\sqrt{2^{x+1}} > \cancel{17} \cdot 2 \sqrt[3]{2^{2(x-3)}}$$

$$2^{\frac{x+1}{2}} > 2 \cdot 2^{\frac{2(x-3)}{3}}$$

$$2^{\frac{x+1}{2}} > 2^{1+\frac{2(x-3)}{3}}$$

$$\frac{x+1}{2} > 1 + \frac{2(x-3)}{3} \rightarrow 2x-6$$

$$\frac{3x+3}{6} > \frac{6+4x-12}{6}$$

$$3x+3 > 6+4x-12$$

$$-x+9 > 0 \qquad -x > -9$$

$$\boxed{x < 9}$$

288 $34\left(\frac{3}{5}\right)^x < 25\left(\frac{9}{25}\right)^x + 9$ $\quad [x < 0 \vee x > 2]$

$$34\left(\frac{3}{5}\right)^x < 25\left(\frac{3}{5}\right)^{2x} + 9 \qquad t = \left(\frac{3}{5}\right)^x$$

$$34t < 25t^2 + 9$$

$$-25t^2 + 34t - 9 < 0$$

$$25t^2 - 34t + 9 > 0 \qquad \frac{\Delta}{4} = 289 - 225 = 64$$

$$t = \frac{17 \pm 8}{25} = \begin{cases} \frac{9}{25} \\ 1 \end{cases}$$

$$t < \frac{9}{25} \quad \vee \quad t > 1$$

$$\left(\frac{3}{5}\right)^x < \frac{9}{25} \quad \vee \quad \left(\frac{3}{5}\right)^x > 1$$

$$\left(\frac{3}{5}\right)^x < \left(\frac{3}{5}\right)^2 \quad \vee \quad \left(\frac{3}{5}\right)^x > \left(\frac{3}{5}\right)^0$$

$$x > 2 \quad \vee \quad x < 0$$

$$\boxed{x < 0 \quad \vee \quad x > 2}$$

294 $\dfrac{-6}{2^x-2}+\dfrac{9}{2^x-1}<0$ $\quad [x<0 \vee 1<x<2]$

$$2^x = t$$

$$\frac{-6}{t-2}+\frac{9}{t-1}<0$$

$$\frac{-6(t-1)+9(t-2)}{(t-2)(t-1)}<0$$

$$\frac{-6t+6+9t-18}{(t-2)(t-1)}<0$$

$$\frac{3t-12}{(t-2)(t-1)}<0$$

$3t-12>0 \qquad t>4$

$t-2>0 \qquad t>2$

$t-1>0 \qquad t>1$

	1		2		4	
−		−		−	0	+
−		−	∄	+		+
−	∄	+		+		+
−	∄	+	∄	−	0	+

$t<1 \quad \vee \quad 2<t<4$

$2^x<1 \quad \vee \quad 2<2^x<2^2$

$$\boxed{x<0 \quad \vee \quad 1<x<2}$$

289 $9\left(\frac{2}{3}\right)^x + 2 + 4\left(\frac{2}{3}\right)^{-x} \leq 0$ [impossibile]

$\left(\frac{2}{3}\right)^x = t$

$$9t + 2 + 4t^{-1} \leq 0$$

$$9t + 2 + \frac{4}{t} \leq 0$$

$$(*) \qquad \frac{9t^2+2t+4}{t} \leq 0$$

$\boxed{\text{NUM.}}$ $9t^2+2t+4 > 0 \quad \forall t$

$$\frac{\Delta}{4} = 1 - 36 < 0$$

$\boxed{\text{DEN.}}$ $t > 0$

		0	
NUM.	+		+
DEN.	−	∄	+
	−	∄	+

$t < 0$ soluzione della diseq. $(*)$

$$\Downarrow$$

$\left(\frac{2}{3}\right)^x < 0$ IMPOSSIBILE

OSSERVAZIONE

Data la disequazione $\frac{9t^2+2t+4}{t} \leq 0$, dato che $t = \left(\frac{2}{3}\right)^x > 0 \ \forall x$, posso semplificare il denominatore $\Rightarrow$ $9t^2+2t+4 \leq 0$

Risolvendo, dato che $\Delta < 0$, trovo ancora IMPOSSIBILE

330 $\dfrac{5^{\frac{4}{3}x+3}}{\sqrt{49^{x+2}}} \leq \dfrac{7 \cdot \sqrt[3]{25^x}}{\sqrt[3]{7^x}}$ $\left[x \geq -\dfrac{9}{2}\right]$

$$\frac{5^{\frac{4}{3}x+3}}{7^{\frac{\cancel{2}(x+2)}{\cancel{2}}}} \leq \frac{7 \cdot 5^{\frac{2x}{3}}}{7^{\frac{x}{3}}}$$

$$\frac{5^{\frac{4}{3}x+3}}{5^{\frac{2x}{3}}} \leq \frac{7 \cdot 7^{x+2}}{7^{\frac{x}{3}}}$$

$$5^{\frac{4}{3}x+3-\frac{2x}{3}} \leq 7^{1+x+2-\frac{x}{3}}$$

$$5^{\frac{4x+9-2x}{3}} \leq 7^{\frac{9+3x-x}{3}}$$

$$5^{\frac{2x+9}{3}} \leq 7^{\frac{2x+9}{3}}$$

$$\left(\frac{5}{7}\right)^{\frac{2x+9}{3}} \leq 1 \leftarrow \left(\frac{5}{7}\right)^0$$

inverto la dis. perché $\frac{5}{7} < 1$

$$\frac{2x+9}{3} \geq 0$$

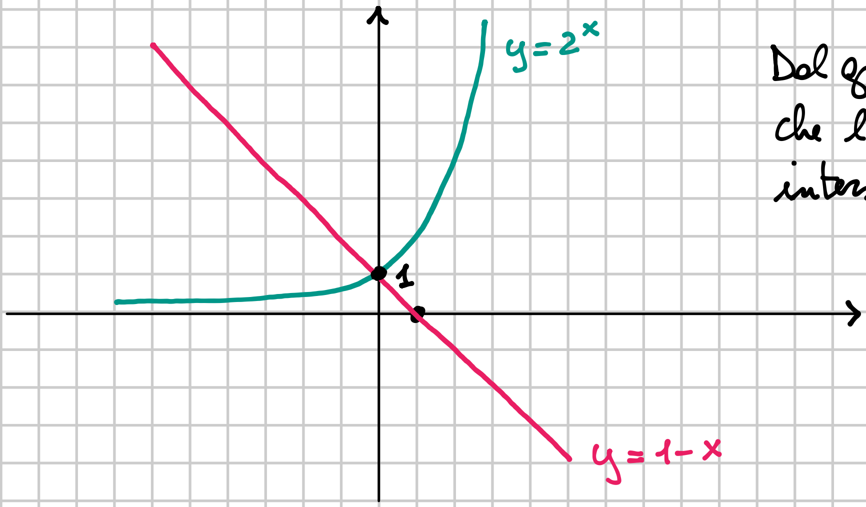
$$2x+9 \geq 0$$

$$\boxed{x \geq -\frac{9}{2}}$$

363 $2^x - 1 = -x$

$2^x = 1 - x$

$$\begin{cases} y = 2^x & \text{curva esponenziale} \\ y = 1 - x & \text{retta} \end{cases}$$



Dal grafico si vede che l'UNICO punto di intersezione è $(0,1)$

$\Downarrow$

$\boxed{x = 0}$