"Compattare"

138 $\log 20x + \log x - 2\log\sqrt{x^2+x} - 1 =$

$= \log(20x^2) - \log\left(\sqrt{x^2+x}\right)^2 - \log 10 =$

$= \log(20x^2) - \left[\log(x^2+x) + \log 10\right] =$

$= \log(20x^2) - \log\left[10(x^2+x)\right] =$

$= \log\frac{2\cancel{0}x^2}{1\cancel{0}(x^2+x)} = \log\frac{2x^{\cancel{2}}}{\cancel{x}(x+1)} = \log\frac{2x}{x+1}$

139 $2 + \log_2 24 + \log_2 3 - \left(2\log_2 2 - \log_2\frac{1}{6}\right) =$

$= 2\cdot\cancel{\overbrace{\log_2 2}^{1}} + \log_2(24\cdot 3) - 2\cancel{\log_2 2} + \log_2\frac{1}{6} =$

$= \log_2\left(\cancelto{4}{24}\cdot 3\cdot\frac{1}{\cancel{6}}\right) = \log_2 12$

154 $\log_3 8 \cdot \log_4 27 =$

$\log_a x = \frac{\log_n x}{\log_n a}$

$= \log_3 8 \cdot \frac{\log_3 27}{\log_3 4} = \log_3 2^3 \cdot \frac{3}{\log_3 2^2} =$

$= 3\cdot\cancel{\log_3 2}\cdot\frac{3}{2\cdot\cancel{\log_3 2}} = \frac{9}{2}$

160 $\dfrac{\log_3 12 - \log_9 4}{\log_{\frac{1}{3}} 6} =$

$$= \frac{\log_3 12 - \dfrac{\log_3 4}{\log_3 9}}{\dfrac{\log_3 6}{\underbrace{\log_3 \frac{1}{3}}_{-1}}} = \frac{\log_3 12 - \frac{1}{2}\log_3 4}{-\log_3 6} =$$

$$= \frac{\log_3 12 - \log_3 4^{\frac{1}{2}}}{-\log_3 6} = \frac{\log_3 12 - \log_3 2}{-\log_3 6} = \frac{\log_3 \frac{12}{2}}{-\log_3 6} =$$

$$= \frac{\log_3 6}{-\log_3 6} = -1$$

285 $3 - \log_2(x^2 - 2x) = 0$ $[-2; 4]$

$$-\log_2(x^2-2x) = -3$$

$$\log_2(x^2-2x) = 3 \cdot \underbrace{\log_2 2}_{1}$$

$$\log_2(x^2-2x) = \log_2 2^3$$

$$x^2-2x = 2^3$$

$$x^2-2x-8=0$$

$$(x-4)(x+2)=0 \rightarrow x=4, \quad x=-2$$

entrambe accettabili per C.E.

C.E. $x^2-2x>0$

$x(x-2)>0$

$x<0 \vee x>2$

$$\boxed{x=4 \vee x=-2}$$

305 $\log x - \log(x+1) = \log 2 - \log 5$ $\left[\frac{2}{3}\right]$

$$\log\frac{x}{x+1} = \log\frac{2}{5}$$

$$\frac{x}{x+1} = \frac{2}{5}$$

$$5x = 2(x+1)$$

$$5x = 2x+2$$

$$3x = 2$$

$$x = \frac{2}{3}$$

C.E.

$\begin{cases} x>0 \\ x+1>0 \end{cases} \quad \begin{cases} x>0 \\ x>-1 \end{cases}$

$\Rightarrow x>0$

314 $\log_{\frac{1}{2}}(x^2-4x)+\log_2 2x-1=0$ [5]

$$\log_{\frac{1}{2}}(x^2-4x)+\frac{\log_{\frac{1}{2}} 2x}{\log_{\frac{1}{2}} 2}-\log_{\frac{1}{2}}\frac{1}{2}=0$$

C.E.
$$\begin{cases} x^2-4x>0 \\ 2x>0 \end{cases}$$

$$\log_{\frac{1}{2}}(x^2-4x)-\log_{\frac{1}{2}} 2x=\log_{\frac{1}{2}}\frac{1}{2}$$

$$\begin{cases} x<0 \vee x>4 \\ x>0 \end{cases} \Rightarrow \boxed{x>4}$$ C.E.

$$\log_{\frac{1}{2}}\frac{x^2-4x}{2x}=\log_{\frac{1}{2}}\frac{1}{2}$$

$$\frac{x^2-4x}{\cancel{2}x}=\frac{1}{\cancel{2}}$$

$$\frac{\cancel{x}(x-4)}{\cancel{x}}=1$$

$\boxed{x=5}$ accettabile dopo controllo C.E.