385 $\ln x \cdot \ln x^2 + \ln x^3 - 2 = 0$ $\left[\frac{1}{e^2}; \sqrt{e}\right]$

C.E.

$x > 0$

$$\ln x \cdot 2\ln x + 3\ln x - 2 = 0$$

$$2(\ln x)^2 + 3\ln x - 2 = 0$$

$\ln x = t$

$$2t^2 + 3t - 2 = 0 \qquad \Delta = 9 + 16 = 25$$

$$t = \frac{-3 \pm 5}{4} = \begin{cases} -2 \Rightarrow \ln x = -2 \Rightarrow x = e^{-2} \\ \frac{1}{2} \Rightarrow \ln x = \frac{1}{2} \Rightarrow x = e^{1/2} \end{cases}$$

$$\boxed{x = \frac{1}{e^2} \quad \vee \quad x = \sqrt{e}}$$

398 $\frac{3}{\ln x} + \frac{\ln x}{\ln x + 1} = 2 + \frac{1}{\ln x}$ $[e^{\sqrt{2}}; e^{-\sqrt{2}}]$

C.E.

$$\begin{cases} x > 0 \\ \ln x \neq 0 \Rightarrow x \neq 1 \\ \ln x \neq -1 \Rightarrow x \neq e^{-1} \end{cases}$$

$$\ln x = -1 \cdot \ln e = \ln e^{-1}$$

$$\Downarrow$$

$$x = e^{-1}$$

$$\begin{cases} x > 0 \\ x \neq 1 \\ x \neq \frac{1}{e} \end{cases}$$

$t = \ln x$

$$\frac{3}{t} + \frac{t}{t+1} = 2 + \frac{1}{t}$$

$$\frac{3(t+1) + t^2}{\cancel{t(t+1)}} = \frac{2t(t+1) + t + 1}{\cancel{t(t+1)}}$$

$$\cancel{3t} + 3 + t^2 = 2t^2 + \cancel{2t} + \cancel{t} + 1$$

$$t^2 = 2 \qquad t = \pm\sqrt{2}$$

accettabili entrambe

$$t = \sqrt{2} \Rightarrow \ln x = \sqrt{2} \Rightarrow x = e^{\sqrt{2}}$$

$$t = -\sqrt{2} \Rightarrow \ln x = -\sqrt{2} \Rightarrow x = e^{-\sqrt{2}} = \frac{1}{e^{\sqrt{2}}}$$

$$\boxed{x = e^{\sqrt{2}} \quad \vee \quad x = e^{-\sqrt{2}}}$$

EQUAZIONE ESPONENZIALE

551 $5^x = 9$ $\left[\frac{\log 9}{\log 5}\right]$

1) $5^x = 9 \iff x = \log_5 9 = \frac{\log 9}{\log 5} = 1{,}3652\ldots$

infatti $5^{1{,}3652} = 8{,}9998\ldots \simeq 9$

2) $5^x = 9$ applico a entrambi i membri $\log_5$

$\log_5(5^x) = \log_5 9$

$x = \log_5 9$

3) $5^x = 9$ applico a entrambi i membri $\log$

$\log 5^x = \log 9$

$x \cdot \log 5 = \log 9 \implies x = \frac{\log 9}{\log 5}$

4) l'esponenziale in base a e il logaritmo in base a sono l'uno l'inverso dell'altro. Ciò significa che

$n = a^{\log_a n}$ e $n = \log_a a^n$ $\left(n \in \mathbb{R} \quad \begin{matrix} a>0 \\ a \neq 1 \end{matrix}\right)$

$5^x = 9 \implies 5^x = \underbrace{5^{\log_5 9}}_{\text{ho scritto 9 come potenza di base 5}} \implies x = \log_5 9$

$9 = 5^{\log_5 9}$

564 $5^x \cdot 2^{2x} = 10$ $\left[\dfrac{\log 5 + \log 2}{\log 5 + 2\log 2}\right]$

1) $(5 \cdot 2^2)^x = 10$

$20^x = 10$

$$x = \log_{20} 10 = \frac{\log 10}{\log 20} = \frac{\log(5 \cdot 2)}{\log(5 \cdot 2^2)} = \frac{\overbrace{\log 5 + \log 2}^{1}}{\log 5 + \log 2^2} =$$

$$= \frac{\log 5 + \log 2}{\log 5 + 2\log 2}$$

2) $5^x \cdot 2^{2x} = 10$

applico "brutalmente" log a entrambi i membri

$\log(5^x \cdot 2^{2x}) = \overbrace{\log 10}^{1}$

$\log 5^x + \log 2^{2x} = 1$

$x \log 5 + 2x \log 2 = 1$

$x(\log 5 + 2\log 2) = 1$

$$x = \frac{1}{\log 5 + 2\log 2}$$

482 $\log_4(x-1) \leq -2$ $\left[1 < x \leq \frac{17}{16}\right]$

C.E.

$x - 1 > 0$

$\log_4(x-1) \leq \log_4 4^{-2}$

dato che la base è $4>1$, <u>non</u> si inverte la disuguaglianza

$$\begin{cases} x-1 \leq 4^{-2} \\ x-1 > 0 \end{cases}$$

$$\begin{cases} x \leq 1 + \frac{1}{16} \\ x > 1 \end{cases} \qquad \begin{cases} x \leq \frac{17}{16} \\ x > 1 \end{cases} \qquad \boxed{1 < x \leq \frac{17}{16}}$$

496 $\log_{\frac{1}{3}}(2x+8) \geq \log_{\frac{1}{3}} 6x - 1$ $\left[x \geq \frac{1}{2}\right]$

$$\begin{cases} 2x+8 > 0 \\ 6x > 0 \\ \log_{\frac{1}{3}}(2x+8) \geq \log_{\frac{1}{3}} 6x - \log_{\frac{1}{3}} \frac{1}{3} \end{cases}$$

$$\begin{cases} x > -4 \\ x > 0 \\ \log_{\frac{1}{3}}(2x+8) \geq \log_{\frac{1}{3}} \frac{6x}{\frac{1}{3}} \end{cases} \qquad \begin{cases} x > 0 \\ \log_{\frac{1}{3}}(2x+8) \geq \log_{\frac{1}{3}} 18x \end{cases}$$

DEVO INVERTIRE LA DISUGUAGLIANZA PERCHÉ LA BASE È $\frac{1}{3} < 1$

$$\begin{cases} x > 0 \\ 2x+8 \leq 18x \end{cases}$$

$$\begin{cases} x > 0 \\ -16x \leq -8 \end{cases} \qquad \begin{cases} x > 0 \\ x \geq \frac{1}{2} \end{cases} \Rightarrow \boxed{x \geq \frac{1}{2}}$$

514 $\ln x + \dfrac{2}{\ln x} - 3 \le 0$ $\quad [0 < x < 1 \lor e \le x \le e^2]$

$$\begin{cases} x > 0 \\ \ln x + \dfrac{2}{\ln x} - 3 \le 0 \end{cases}$$

$\ln x = t$ $\qquad t + \dfrac{2}{t} - 3 \le 0$

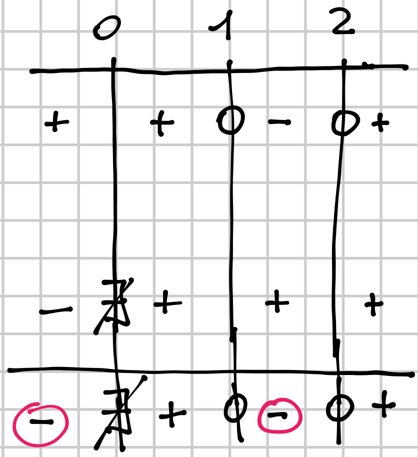
$$\frac{t^2 + 2 - 3t}{t} \le 0$$

$$\frac{t^2 - 3t + 2}{t} \le 0$$

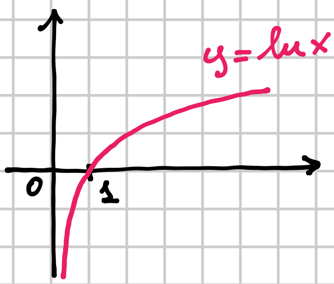
$\boxed{\text{NUM}}$ $\quad t^2 - 3t + 2 > 0 \qquad t < 1 \lor t > 2$

$(t-1)(t-2) > 0$

$\boxed{\text{DEN}}$ $\quad t > 0$



$t < 0 \quad \lor \quad 1 \le t \le 2$



$$\begin{cases} \ln x < 0 \quad \lor \quad 1 \le \ln x \le 2 \\ x > 0 \end{cases}$$

$0 < x < 1 \quad \lor \quad \ln e \le \ln x \le \ln e^2$

$$\boxed{0 < x < 1 \qquad \lor \qquad e \le x \le e^2}$$

$1 \le \ln x \le 2$