508 $\frac{1}{2}\log_x[2(1-x)] + \log_x\sqrt{x} + \frac{1}{4}\log_x x^2 < 2$

$[0 < x < \sqrt{3} - 1]$

C.E.

$$\begin{cases} \cancel{2}(1-x) > 0 \Rightarrow x < 1 \\ x > 0 \\ \boxed{x > 0 \quad x \neq 1} \end{cases}$$

→ CONDIZIONI PER ESSERE BASE DEL LOG.

$\Downarrow$

$0 < x < 1$

$$\begin{cases} \frac{1}{2}\log_x[2(1-x)] + \frac{1}{2}\overbrace{\log_x x}^{1} + \frac{1}{4}\cdot 2\overbrace{\log_x x}^{1} < 2 \\ 0 < x < 1 \end{cases}$$

$$\begin{cases} \frac{1}{2}\log_x[2(1-x)] + \frac{1}{2} + \frac{1}{2} < 2 \\ 0 < x < 1 \end{cases}$$

$\downarrow$

$\frac{1}{2}\log_x[2(1-x)] + 1 < 2$

$\frac{1}{2}\log_x[2(1-x)] < 1$

$$\begin{cases} \log_x[2(1-x)] < 2 \\ 0 < x < 1 \end{cases}$$

← applico a entrambi i membri l'esponenziale in base x, ma dato che $0 < x < 1$, la disuguaglianza si inverte

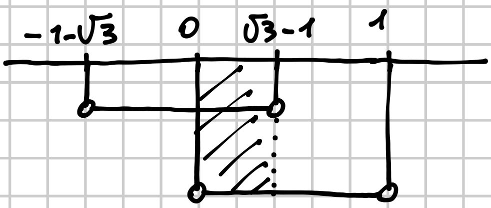
$$\begin{cases} 2(1-x) > x^2 \\ 0 < x < 1 \end{cases}$$

$$\begin{cases} 2 - 2x - x^2 > 0 \\ 0 < x < 1 \end{cases} \qquad \begin{cases} x^2 + 2x - 2 < 0 \\ 0 < x < 1 \end{cases}$$

$\frac{\Delta}{4} = 1 + 2 = 3$

$x = -1 \pm \sqrt{3}$

$$\begin{cases} -1-\sqrt{3} < x < \sqrt{3} - 1 \\ 0 < x < 1 \end{cases}$$



$\boxed{0 < x < \sqrt{3} - 1}$

524 $\log^4 x - 8\log^2 x + 16 > 0$

$\left[x > 0 \land x \neq \dfrac{1}{100} \land x \neq 100\right]$

C.E. $x > 0$

$t = \log^2 x$

$t^2 - 8t + 16 > 0$

$(t-4)^2 > 0$

$t \neq 4$

$\log^2 x \neq 4 \rightarrow (\log x)^2 \neq 4$

$\log x \neq \pm 2$

$$\begin{cases} \log x \neq -2 \quad \land \quad \log x \neq 2 \\ x > 0 \end{cases}$$

$$\begin{cases} x \neq 10^{-2} \quad \land \quad x \neq 10^2 \\ x > 0 \end{cases}$$

$$\boxed{x > 0 \quad \land \quad x \neq \frac{1}{100} \quad \land \quad x \neq 100}$$

562 $3^x + 20 = 9^x$ $\left[\frac{\log 5}{\log 3}\right]$

$3^x + 20 = (3^x)^2$ $t = 3^x$

$t + 20 = t^2$

$t^2 - t - 20 = 0$ $t = \frac{1 \pm 9}{2} =$ $-4 \Rightarrow 3^x = -4$ IMP. (N. ACC.)

$5 \Rightarrow 3^x = 5$

$\Delta = 1 + 80 = 81$

$x = \log_3 5 = \frac{\log 5}{\log 3}$

568 $2^{2x+3} - 25 \cdot 2^x + 3 = 0$ $\left[-3; \frac{\log 3}{\log 2}\right]$

$2^{2x} \cdot 2^3 - 25 \cdot 2^x + 3 = 0$ $t = 2^x$

$8t^2 - 25t + 3 = 0$ $\Delta = 625 - 96 = 529 = 23^2$

$t = \frac{25 \pm 23}{16} =$ $\frac{2}{16} = \frac{1}{8} \Rightarrow 2^x = \frac{1}{8} \Rightarrow 2^x = 2^{-3} \Rightarrow x = -3$

$3 \Rightarrow 2^x = 3 \Rightarrow x = \log_2 3 = \frac{\log 3}{\log 2}$

$$\boxed{x = -3 \vee x = \frac{\log 3}{\log 2}}$$

569 $\frac{2}{5^x} = \frac{3}{7^x}$ $\left[\frac{\log 3 - \log 2}{\log 7 - \log 5}\right]$

1° MODO

$$\frac{7^x}{5^x} = \frac{3}{2} \qquad \left(\frac{7}{5}\right)^x = \frac{3}{2} \qquad x = \log_{\frac{7}{5}} \frac{3}{2} = \frac{\log \frac{3}{2}}{\log \frac{7}{5}} = \frac{\log 3 - \log 2}{\log 7 - \log 5}$$

2° MODO Applico "brutalmente" a entrambi i membri il log

$$\log \frac{2}{5^x} = \log \frac{3}{7^x}$$

$$\log 2 - \log 5^x = \log 3 - \log 7^x$$

$$\log 2 - x \log 5 = \log 3 - x \log 7$$

$$x \log 7 - x \log 5 = \log 3 - \log 2$$

$$x (\log 7 - \log 5) = \log 3 - \log 2$$

$$x = \frac{\log 3 - \log 2}{\log 7 - \log 5}$$

555 $\sqrt[3]{7^x} = 5$ $\left[\frac{3\log 5}{\log 7}\right]$

1° MODO $7^{\frac{x}{3}} = 5 \qquad \frac{x}{3} = \log_7 5 \qquad x = 3\log_7 5 = 3\frac{\log 5}{\log 7}$

2° MODO $\log 7^{\frac{x}{3}} = \log 5$

$$\frac{x}{3}\log 7 = \log 5 \qquad x = 3\frac{\log 5}{\log 7}$$

522 $\dfrac{\log_2 x}{\log_{\frac{1}{2}} 2x + 2} > 0$ $[1 < x < 2]$

$\boxed{C.E. \quad x > 0}$ NON DIMENTICARLA!!!

$$\frac{\log_2 x}{\frac{\log_2 2x}{\underbrace{\log_2 \frac{1}{2}}_{-1}} + 2} > 0$$

$$\frac{\log_2 x}{-\log_2 2x + 2} > 0$$

$$\frac{\log_2 x}{-(\log_2 2 + \log_2 x) + 2} > 0$$

$$\frac{\log_2 x}{-1 - \log_2 x + 2} > 0$$

$$\frac{\log_2 x}{1 - \log_2 x} > 0 \qquad t = \log_2 x$$

$$\frac{t}{1 - t} > 0$$

$N > 0 \qquad t > 0$

$D > 0 \qquad 1 - t > 0 \qquad t < 1$

	0		1	
N	−	+	+	
D	+	+	−	
	−	+	−	

$$0 < t < 1$$

$$\begin{cases} 0 < \log_2 x < 1 \\ x > 0 \quad C.E. \end{cases} \qquad \begin{cases} 2^0 < 2^{\log_2 x} < 2^1 \\ x > 0 \end{cases}$$

$$\begin{cases} 1 < x < 2 \\ x > 0 \end{cases}$$

$$\boxed{1 < x < 2}$$