631 $\dfrac{3}{10^x-2}-\dfrac{1}{10^x+2}>1-\dfrac{2}{10^x+2}$ $\quad [\log 2 < x < \log(2+\sqrt{12})]$

$t = 10^x$

$$\frac{3}{t-2}-\frac{1}{t+2}-1+\frac{2}{t+2}>0$$

$$\frac{3t+6-t+2-t^2+\cancel{4}+2t-\cancel{4}}{(t-2)(t+2)}>0$$

$$\frac{-t^2+4t+8}{(t-2)(t+2)}>0 \qquad \frac{\overset{\boxed{N}}{t^2-4t-8}}{\underset{\boxed{D_1}}{(t-2)}\underset{\boxed{D_2}}{(t+2)}}<0$$

$N>0 \quad t^2-4t-8>0 \quad \dfrac{\Delta}{4}=4+8=12 \quad t=2\pm\sqrt{12}=2\pm2\sqrt{3}$

$t<2-2\sqrt{3} \ \vee \ t>2+2\sqrt{3}$

$D_1>0 \quad t-2>0 \quad t>2$

$D_2>0 \quad t+2>0 \quad t>-2$

	-2	$2-2\sqrt{3}$	2	$2+2\sqrt{3}$	
N	+	+	−	−	+
D_1	−	−	−	+	+
D_2	−	+	+	+	+
	+	−	+	−	+

$-2<t<2-2\sqrt{3} \quad \vee \quad 2<t<2+2\sqrt{3}$

$\underbrace{-2<10^x}_{\text{VALE } \forall x}<\underbrace{2-2\sqrt{3}}_{<0} \quad \vee \quad 2<10^x<2+2\sqrt{3}$

IMPOSSIBILE
(per la 2ª disuguag.)

$\Downarrow$

$$\boxed{\log 2 < x < \log(2+2\sqrt{3})}$$

533 $\log_{\frac{1}{2}} \frac{|x-2|}{x} < -1 + \log_2 x$ $[x > 4]$

C.E.

$x > 0 \wedge x \neq 2$

$$\frac{\log_2 \frac{|x-2|}{x}}{\underbrace{\log_2 \frac{1}{2}}_{-1}} < -1 + \log_2 x$$

$$-\log_2 \frac{|x-2|}{x} < -1 + \log_2 x$$

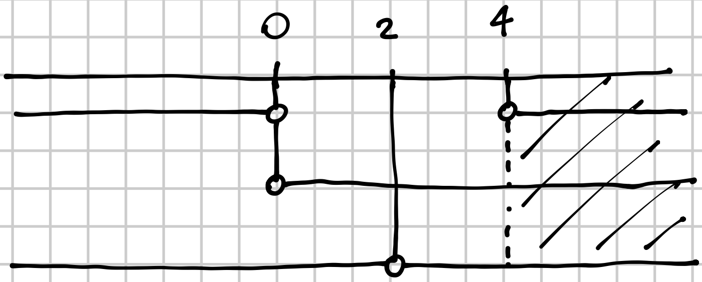
$$\log_2 \frac{|x-2|}{x} > 1 - \log_2 x$$

$$\log_2 \frac{|x-2|}{x} > \log_2 2 - \log_2 x$$

$$\log_2 |x-2| - \cancel{\log_2 x} > \log_2 2 - \cancel{\log_2 x}$$

$$\begin{cases} |x-2| > 2 \\ x > 0 \\ x \neq 2 \end{cases} \qquad \begin{cases} x-2 < -2 \quad \vee \quad x-2 > 2 \\ x > 0 \\ x \neq 2 \end{cases}$$

$$\begin{cases} x < 0 \quad \vee \quad x > 4 \\ x > 0 \\ x \neq 2 \end{cases}$$



$\boxed{x > 4}$

596 $3^{x+1} - 2 \cdot 3^x + 3^{x+2} = 5^{x-1}$ $\left[\dfrac{2\log 5 + \log 2}{\log 5 - \log 3}\right]$

$$3^x \cdot 3^1 - 2 \cdot 3^x + 3^x \cdot 3^2 = 5^x \cdot 5^{-1}$$

$$3^x(3 - 2 + 9) = \frac{1}{5} \cdot 5^x$$

$$3^x \cdot 10 = \frac{1}{5} \cdot 5^x$$

$$3^x = \frac{1}{50} 5^x$$

$$\log 3^x = \log\left(\frac{1}{50} \cdot 5^x\right)$$

$$x \log 3 = \log \frac{1}{50} + x \log 5$$

$$x(\log 5 - \log 3) = -\log \frac{1}{50}$$

$$\vdots$$

$$x = \frac{\log 50}{\log 5 - \log 3} = \dots$$

$$\left(\frac{3}{5}\right)^x = \frac{1}{50}$$

$$x = \log_{\frac{3}{5}} \frac{1}{50} = \frac{\log \frac{1}{50}}{\log \frac{3}{5}} =$$

$$= \frac{\log 1 - \log 50}{\log 3 - \log 5} = \frac{-\log(5^2 \cdot 2)}{\log 3 - \log 5}$$

$$= \frac{-\log 5^2 - \log 2}{\log 3 - \log 5} = \boxed{\frac{2\log 5 + \log 2}{\log 5 - \log 3}}$$

537 $\log_6 \sqrt{x^2 - 2x} < \log_6 |x| - \frac{1}{2}$ $\left[2 < x < \frac{12}{5}\right]$

C.E. $\begin{cases} x \neq 0 \\ x^2 - 2x > 0 \end{cases}$

$\log_6 \sqrt{x^2-2x} < \log_6 |x| - \log_6 6^{\frac{1}{2}}$

$\begin{cases} x \neq 0 \\ x < 0 \vee x > 2 \end{cases}$

$\Downarrow$

$x < 0 \vee x > 2$

$\log_6 \sqrt{x^2-2x} < \log_6 \frac{|x|}{\sqrt{6}}$

$\begin{cases} \sqrt{x^2-2x} < \frac{|x|}{\sqrt{6}} \\ x < 0 \vee x > 2 \end{cases}$ $\begin{cases} x^2-2x < \frac{x^2}{6} \\ x < 0 \vee x > 2 \end{cases}$

$6x^2 - 12x < x^2$

$5x^2 - 12x < 0$

$x(5x-12) < 0$

$x = 0 \quad x = \frac{12}{5}$

radici del polinomio al 1° membro

$\begin{cases} 0 < x < \frac{12}{5} \\ x < 0 \vee x > 2 \end{cases} \Rightarrow \boxed{2 < x < \frac{12}{5}}$