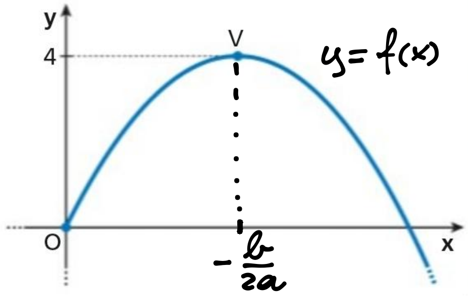
LEGGI IL GRAFICO

84 L'arco di parabola in figura ha equazione

$$f(x) = \frac{1}{k}x\left(-\frac{1}{k}x + 2k\right), \quad x \geq 0.$$



a. Determina il valore di k.

b. Considera la funzione $h(x) = \begin{cases} g(x) & \text{se } x < 0 \\ f(x) & \text{se } x \geq 0 \end{cases}$.

Determina g in modo che h sia una funzione dispari.

c. Effettua una restrizione del dominio in modo che $h(x)$ sia invertibile e determina la funzione inversa h^{-1} sia graficamente sia algebricamente.

$\left[\text{a) } k = \pm 2; \text{ b) } g(x) = \frac{1}{4}x^2 + 2x; \text{ c) su } I = [-4; 4], h^{-1}(x) = \begin{cases} 2\sqrt{x+4} - 4 & \text{se } -4 \leq x < 0 \\ 4 - 2\sqrt{4-x} & \text{se } 0 \leq x \leq 4 \end{cases}\right]$

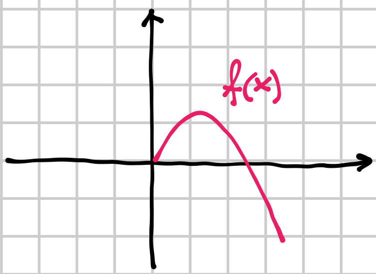
a) $x \geq 0$

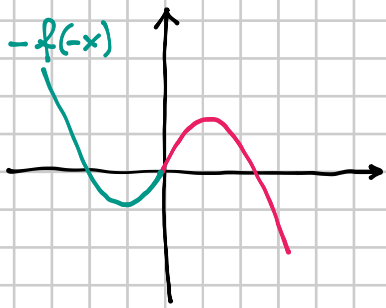
$f(x) = -\frac{1}{k^2}x^2 + 2x \qquad x_V = -\frac{b}{2a} = -\frac{\cancel{2}}{\cancel{2}\left(-\frac{1}{k^2}\right)} = k^2$

Per trovare K pongo $f(k^2) = 4$

$-\frac{1}{k^2}k^4 + 2k^2 = 4 \qquad -k^2 + 2k^2 = 4 \qquad k^2 = 4 \qquad \boxed{k = \pm 2}$

b) $f(x) = -\frac{1}{4}x^2 + 2x \qquad x \geq 0$

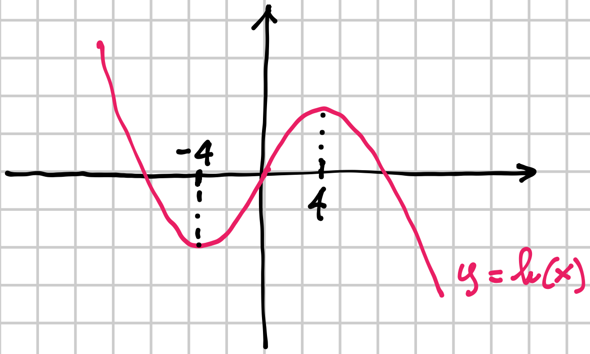




$h(x) = \begin{cases} f(x) & x \geq 0 \\ -f(-x) & x < 0 \end{cases} = \begin{cases} -\frac{1}{4}x^2 + 2x & x \geq 0 \\ \frac{1}{4}x^2 + 2x & x < 0 \end{cases} \Rightarrow g(x) = \frac{1}{4}x^2 + 2x$

$f(-x) = -\frac{1}{4}x^2 - 2x$

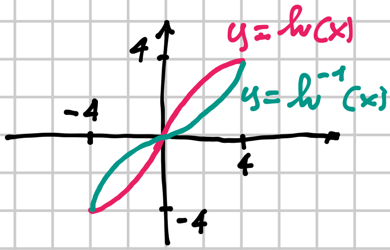
c)



La funzione è invertibile negli intervalli in cui è strettamente crescente o decrescente, cioè $(-\infty,-4]$, $[-4,4]$, $[4,+\infty)$

Scegliamo l'intervallo $[-4,4]$

$$h|_{[-4,4]}(x) = \begin{cases} \frac{1}{4}x^2+2x & -4 \le x \le 0 \\ -\frac{1}{4}x^2+2x & 0 \le x \le 4 \end{cases}$$



$y = \frac{1}{4}x^2+2x \implies 4y = x^2+8x \qquad x^2+8x-4y=0$

$\underline{-4 \le x \le 0}$

$x^2+8x+16-16-4y=0$

$(x+4)^2-16-4y=0$

$(x+4)^2 = 16+4y$

x+4 deve essere positivo perché x è compreso tra -4 e 0

$x+4 = +\sqrt{16+4y} \qquad x = -4+2\sqrt{4+y}$

$$\boxed{y = -4+2\sqrt{4+x}}$$

Oss. Si poteva anche risolvere $x^2+8x-4y=0$ come equazione di 2° grado

$-4 \le x \le 0 \qquad \frac{\Delta}{4} = 16+4y \implies x = -4 \pm \sqrt{16+4y}$ poi scelgo + per lo stesso motivo

$y = -\frac{1}{4}x^2 + 2x \qquad 0 \leq x \leq 4$

$4y = -x^2 + 8x$

$x^2 - 8x + 4y = 0 \qquad x^2 - 8x + 16 - 16 + 4y = 0 \qquad (x-4)^2 = 16 - 4y$

$\underbrace{x-4}_{} = -\sqrt{16-4y}$

deve essere **negativo**
perché $0 \leq x \leq 4$

$x = 4 - \sqrt{16-4y}$

$y = 4 - 2\sqrt{4-x}$

$$h^{-1}(x) = \begin{cases} -4 + 2\sqrt{4+x} & -4 \leq x \leq 0 \\ 4 - 2\sqrt{4-x} & 0 \leq x \leq 4 \end{cases}$$

89 Considera la funzione

$$y = f(x) = 2\left|\log_2 x\right| + \log_2 2x - 2.$$

a. Trova il dominio, studia il segno di $f(x)$ e disegna il grafico di $f(x)$.

b. La funzione è monotòna?
È invertibile?
Se non lo è in tutto il suo dominio, effettua una restrizione, trova $f^{-1}(x)$ e mostra che $f(f^{-1}(x)) = x$.

c. Disegna i grafici di $y = f(x) + 1$ e di $y = f(x+1)$.

$\left[\text{a) } D: x > 0;\ f(x) > 0 \text{ per } 0 < x < \frac{1}{2} \vee x > \sqrt[3]{2};\ \text{b) per } x \geq 1,\ f^{-1}(x) = 2^{\frac{x+1}{3}}\right]$

a) DOMINIO $\quad x > 0 \qquad D = (0, +\infty)$

$$|\log_2 x| = \begin{cases} \log_2 x & \text{se } x \geq 1 \\ -\log_2 x & \text{se } 0 < x < 1 \end{cases}$$

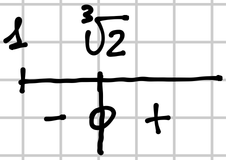
$$\log_2(2x) = \log_2 2 + \log_2 x = 1 + \log_2 x$$

$$f(x) = \begin{cases} 2\log_2 x + 1 + \log_2 x - 2 & \text{se } x \geq 1 \\ -2\log_2 x + 1 + \log_2 x - 2 & \text{se } 0 < x < 1 \end{cases}$$

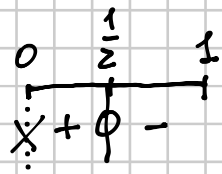
$$f(x) = \begin{cases} 3\log_2 x - 1 & \text{se } x \geq 1 \\ -\log_2 x - 1 & \text{se } 0 < x < 1 \end{cases}$$

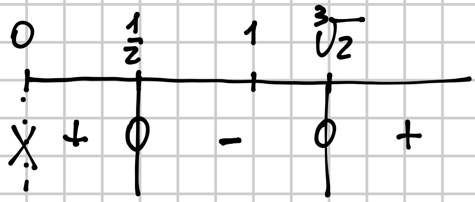
SEGNO

$$\begin{cases} 3\log_2 x - 1 > 0 \\ x \geq 1 \end{cases} \qquad \begin{cases} \log_2 x > \frac{1}{3} \\ x \geq 1 \end{cases} \qquad \begin{cases} x > 2^{\frac{1}{3}} = \sqrt[3]{2} \\ x \geq 1 \end{cases}$$

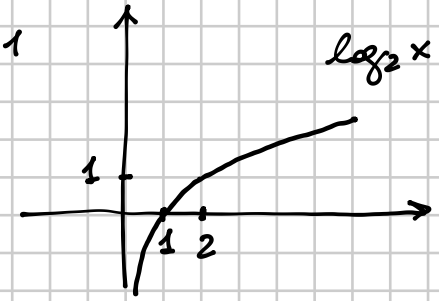


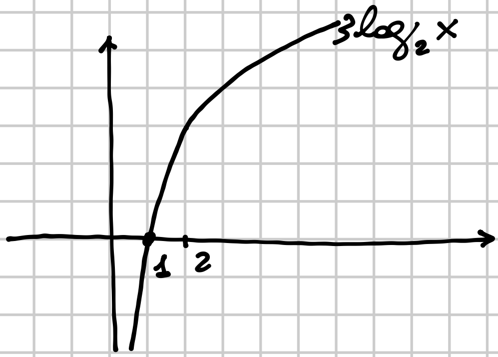
$$\begin{cases} -\log_2 x - 1 > 0 \\ 0 < x < 1 \end{cases} \qquad \begin{cases} \log_2 x < -1 \\ 0 < x < 1 \end{cases} \qquad \begin{cases} x < 2^{-1} = \frac{1}{2} \\ 0 < x < 1 \end{cases}$$

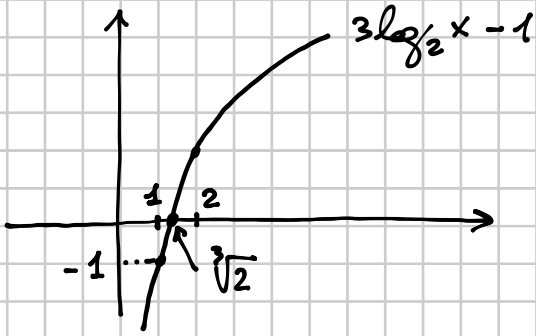




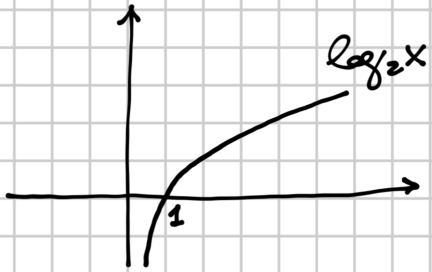
$3\log_2 x - 1$

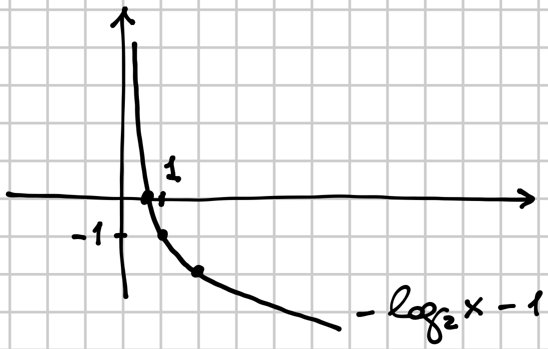




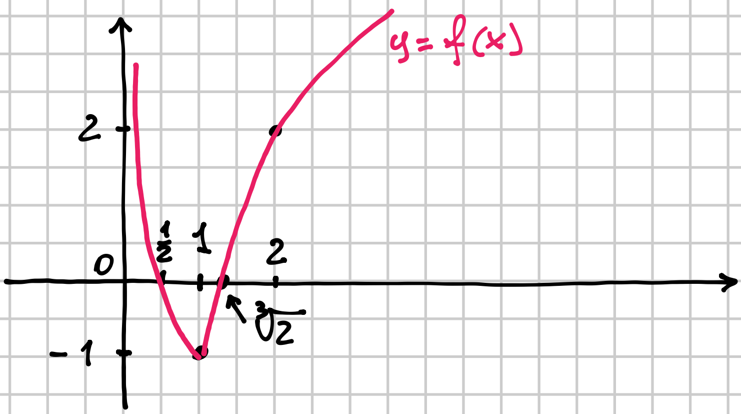


$-\log_2 x - 1$





$y = f(x)$



b) La funzione <u>NON</u> è monotòna, ma è strettamente decrescente nell'intervallo $(0,1]$ e strettamente crescente nell'intervallo $[1,+\infty)$.
Dunque (la restrizione della funzione) è invertibile in ciascuno di questi due intervalli (presi separatamente)

Prendiamo la restrizione di f a $[1,+\infty)$

$$f(x) = 3\log_2 x - 1 \qquad x \geq 1$$

$$y = 3\log_2 x - 1$$

$$y+1 = 3\log_2 x$$

$$\frac{y+1}{3} = \log_2 x \implies x = 2^{\frac{y+1}{3}} \implies y = 2^{\frac{x+1}{3}}$$

$$f^{-1}(x) = 2^{\frac{x+1}{3}} \qquad \text{dom}\, f^{-1} = [-1,+\infty)$$

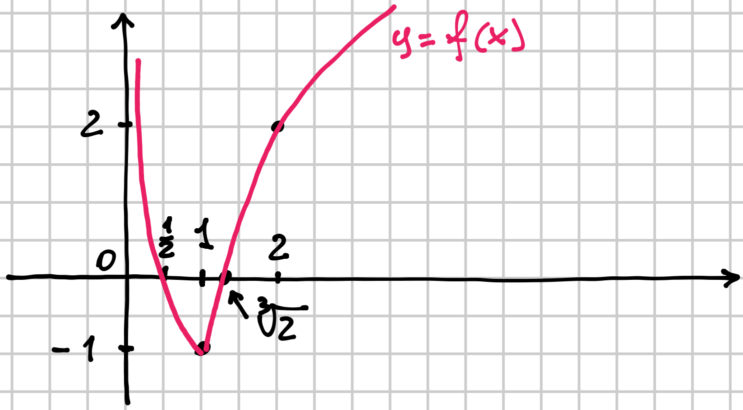
$$f : [1,+\infty) \to [-1,+\infty) \qquad f(x) = 3\log_2 x - 1$$

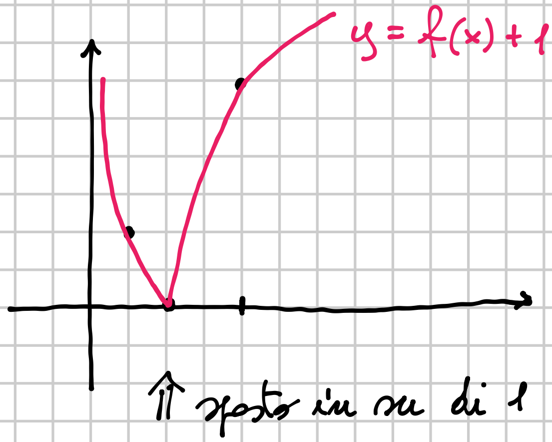
$$f^{-1} : [-1,+\infty) \to [1,+\infty) \qquad f^{-1}(x) = 2^{\frac{x+1}{3}}$$

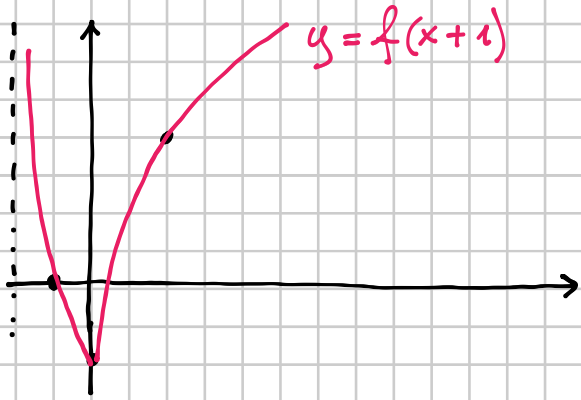
$$f(f^{-1}(x)) = f\left(2^{\frac{x+1}{3}}\right) = 3\log_2\left(2^{\frac{x+1}{3}}\right) - 1 = \cancel{3}\,\frac{x+1}{\cancel{3}} - 1 =$$

$$= x+1-1 = x$$

c)

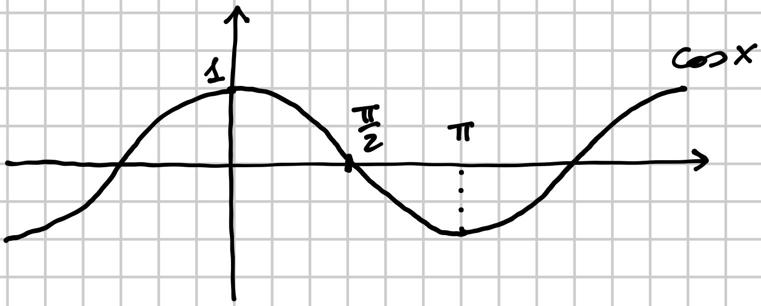


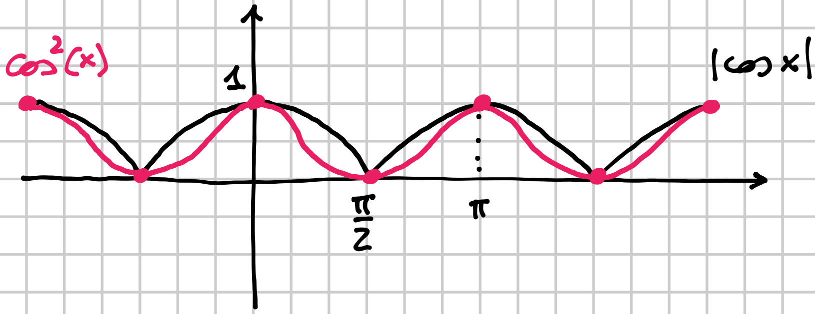


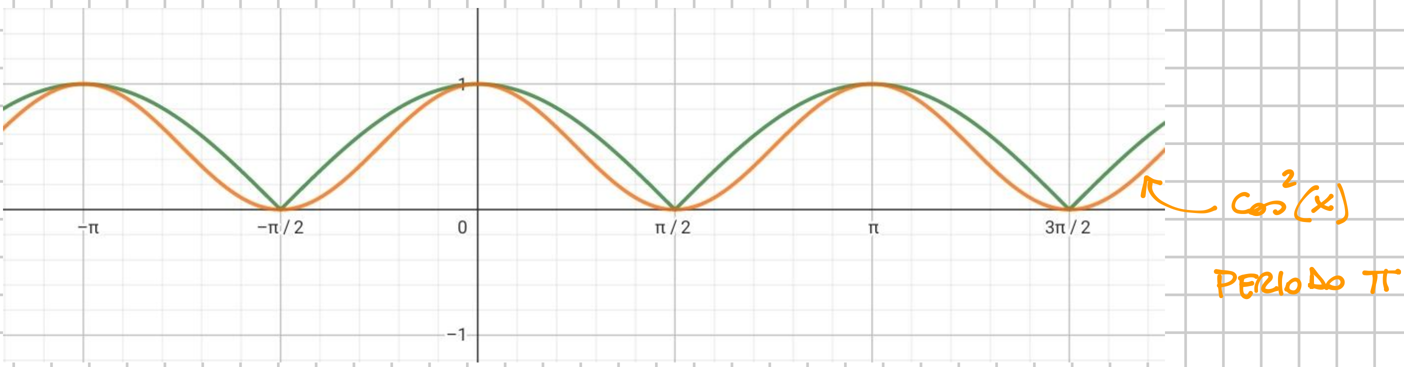


PREMESSA

Come è fatto $\cos^2 x$?







STUDIO

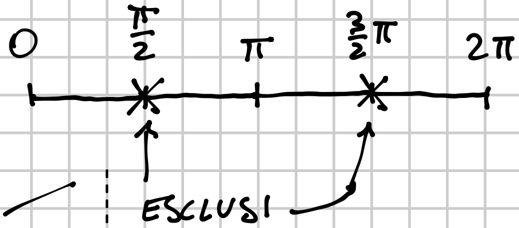
288 $y = \dfrac{2\sin x - 1}{\cos^2 x}$ $\left[\dfrac{\pi}{6} + 2k\pi < x < \dfrac{5}{6}\pi + 2k\pi\right]$

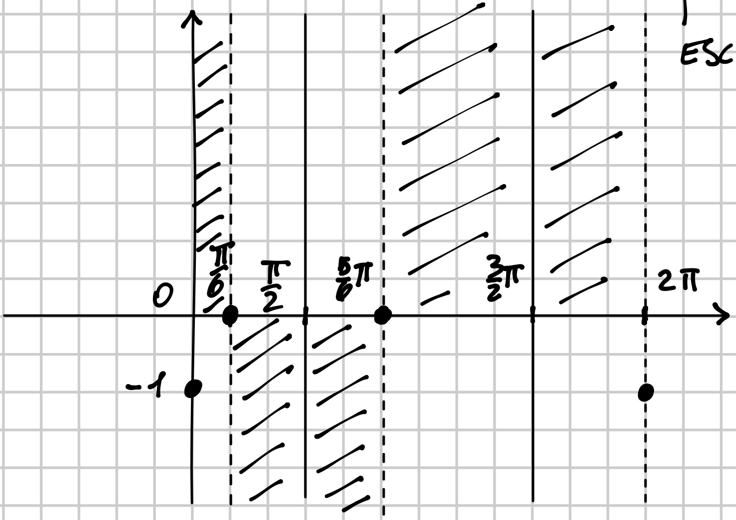
PERIODO 2π

↳ la studio solo in $[0, 2\pi]$, poi aggiungo la periodicità

1) DOMINIO

$\cos x \neq 0 \Rightarrow x \neq \dfrac{\pi}{2} + k\pi$

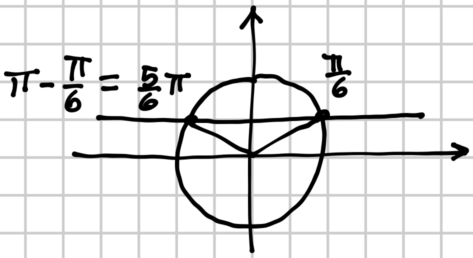




2) INT. ASSI

$\dfrac{2\sin x - 1}{\cos^2 x} = 0 \Rightarrow 2\sin x - 1 = 0 \qquad \sin x = \dfrac{1}{2} \qquad x = \dfrac{\pi}{6} \vee x = \dfrac{5}{6}\pi$

INT. ASSE X



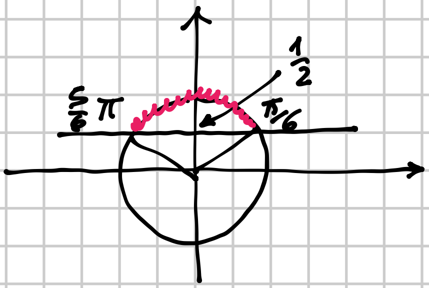
INT. ASSE y

$\begin{cases} x = 0 \\ y = \dfrac{2\sin x - 1}{\cos^2 x} \end{cases} \qquad \begin{cases} x = 0 \\ y = -1 \end{cases} \qquad (0, -1) \quad (2\pi, -1)$

3) SEGNO

$$\frac{2\sin x - 1}{\underbrace{\cos^2 x}_{}} > 0 \Rightarrow 2\sin x - 1 > 0 \qquad \sin x > \frac{1}{2} \qquad \frac{\pi}{6} < x < \frac{5}{6}\pi$$

sempre > 0
dove esiste (nel dominio)



	0		$\frac{\pi}{6}$		$\frac{\pi}{2}$		$\frac{5}{6}\pi$		π		$\frac{3}{2}\pi$		2π
$2\sin x - 1$		$-$	0	$+$		$+$	0	$-$		$-$		$-$	
$\cos^2 x$		$+$		$+$	$\nexists$	$+$		$+$		$+$	$\nexists$	$+$	
		$-$	0	$+$	$\nexists$	$+$	0	$-$		$-$	$\nexists$	$-$	

Il dominio lungo tutta la retta è $\mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi \right\}$