ESERCIZI SUI LIMITI DI SUCCESSIONI

1) $$3 - \frac{2}{5n+1}$$

$$\lim_{n \to +\infty} \left(3 - \frac{2}{5n+1}\right) = 3 - \frac{2}{+\infty} = 3 - 0 = 3$$

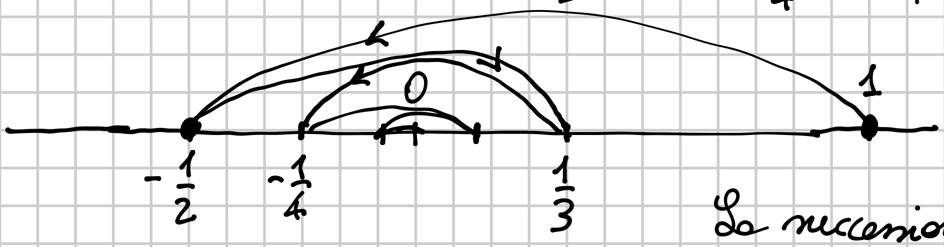
2) $\lim_{n \to +\infty} \frac{(-1)^n}{n+1}$ OSSERVIAMO che $\lim_{n \to +\infty} (-1)^n$ NON ESISTE

perché $(-1)^n = \begin{cases} 1 & \text{se } n \text{ pari} \\ -1 & \text{se } n \text{ dispari} \end{cases}$

$a_n = \frac{(-1)^n}{n+1}$

quindi $(-1)^n$ oscilla tra 1 e -1

$a_0 = 1 \quad a_1 = -\frac{1}{2} \quad a_2 = \frac{1}{3} \quad a_3 = -\frac{1}{4} \quad a_4 = \frac{1}{5} \quad a_5 = -\frac{1}{6} \quad \dots$



La successione salta da valori positivi a negativi avvicinandosi sempre più a 0

$$\lim_{n \to +\infty} \frac{(-1)^n}{n+1} = 0$$

Teorema dei due carabinieri. *Siano $\{a_n\}$, $\{c'_n\}$ e $\{c''_n\}$ tre successioni reali tali che*

$$c'_n \leq a_n \leq c''_n \quad \forall n.$$

Se le successioni $\{c'_n\}$ e $\{c''_n\}$ convergono a uno stesso limite L, allora anche $\{a_n\}$ converge a L.

$a_n = \frac{(-1)^n}{n+1} \qquad c'_n = \frac{-1}{n+1} \qquad c_n = \frac{1}{n+1}$

$$\frac{-1}{n+1} \leq \frac{(-1)^n}{n+1} \leq \frac{1}{n+1}$$

$$\frac{-1}{n+1} \to 0 \qquad \frac{1}{n+1} \to 0$$

quindi anche quello in mezzo tende a 0

3) $$\lim_{n \to +\infty} \frac{n^2 - 5n + 1}{4n^2 - 3n + 5} = \frac{+\infty}{+\infty} \quad \text{F.I.}$$

$$= \lim_{n \to +\infty} \frac{\cancel{n^2}\left(1 - \frac{5}{n} + \frac{1}{n^2}\right)}{\cancel{n^2}\left(4 - \frac{3}{n} + \frac{5}{n^2}\right)} = \frac{1}{4}$$

$\frac{5}{n} \to 0$, $\frac{1}{n^2} \to 0$, $\frac{3}{n} \to 0$, $\frac{5}{n^2} \to 0$

4) $$\lim_{n \to +\infty} \frac{3n - 7}{8n^2 + 4n + 5} = \lim_{n \to +\infty} \frac{\cancel{n}\left(3 - \frac{7}{n}\right)}{n^{\cancel{2}}\left(8 + \frac{4}{n} + \frac{5}{n^2}\right)} = \frac{3}{+\infty} = 0$$

$\frac{7}{n} \to 0$, $\frac{4}{n} \to 0$, $\frac{5}{n^2} \to 0$

5) $$\lim_{n \to +\infty} \frac{2n^3 - 5n + 1}{3n^2 + n - 2} = \lim_{n \to +\infty} \frac{n^{\cancel{3}}\left(2 - \frac{5}{n^2} + \frac{1}{n^3}\right)}{\cancel{n^2}\left(3 + \frac{1}{n} - \frac{2}{n^2}\right)} = \frac{+\infty \cdot 2}{3} =$$

$$= +\infty$$

$\frac{5}{n^2} \to 0$, $\frac{1}{n^3} \to 0$, $\frac{1}{n} \to 0$, $\frac{2}{n^2} \to 0$