13. $\dfrac{4n^2+1}{3n^2+n-4}$ $\left[\dfrac{4}{3}\right]$

14. $\dfrac{n+1}{3n^2-4}$ $[0]$

15. $\dfrac{4n-1}{n+1}$ $[4]$

16. $\dfrac{n^3-4n+1}{2-3n}$ $[-\infty]$

17. $\dfrac{n^2-n}{3n+4}$ $[+\infty]$

18. $\dfrac{5+n-n^2+n^3}{1-2n^3}$ $\left[-\dfrac{1}{2}\right]$

$$\lim_{n\to+\infty}\frac{4n^2+1}{3n^2+n-4}=\lim_{n\to+\infty}\frac{\cancel{n^2}\left(4+\frac{1}{n^2}\right)}{\cancel{n^2}\left(3+\frac{1}{n}-\frac{4}{n^2}\right)}=\frac{4}{3}$$

$$\lim_{n\to+\infty}\frac{\cancel{n}\left(1+\frac{1}{n}\right)}{n^{\cancel{2}}\left(3-\frac{4}{n^2}\right)}=\frac{1}{+\infty}=0$$

$$16]\quad \lim_{n\to+\infty}\frac{n^3-4n+1}{2-3n}=\lim_{n\to+\infty}\frac{n^{\cancel{3}\,2}\left(1-\frac{4}{n^2}+\frac{1}{n^3}\right)}{\cancel{n}\left(\frac{2}{n}-3\right)}=\frac{(+\infty)^2}{-3}=\frac{+\infty}{-3}=-\infty$$

$$17]\quad \lim_{n\to+\infty}\frac{n^2-n}{3n+4}=\lim_{n\to+\infty}\frac{n^{\cancel{2}}\left(1-\frac{1}{n}\right)}{\cancel{n}\left(3+\frac{4}{n}\right)}=\frac{+\infty\cdot 1}{3}=+\infty$$

$$\lim_{n\to+\infty}\left(2n-3\sqrt{n}\right)=+\infty-\infty \qquad \text{F.I.}$$

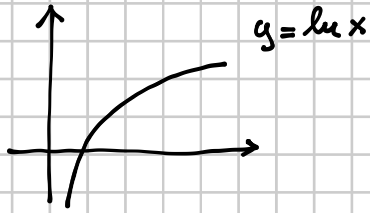
$$=\lim_{n\to+\infty} n\left(2-3\frac{\sqrt{n}}{\underset{\sqrt{n}\cdot\sqrt{n}}{n}}\right)=\lim_{n\to+\infty}\underset{+\infty}{n}\overbrace{\left(\underset{2}{2}-\underset{0}{\frac{3}{\sqrt{n}}}\right)}=+\infty\cdot(2-0)=$$

$$=+\infty\cdot 2=+\infty$$

$$\lim_{n\to+\infty}\underbrace{(1-3\sqrt{n})}_{-\infty}\underbrace{\frac{n-1}{n+2}}_{1}=-\infty\cdot 1=-\infty$$

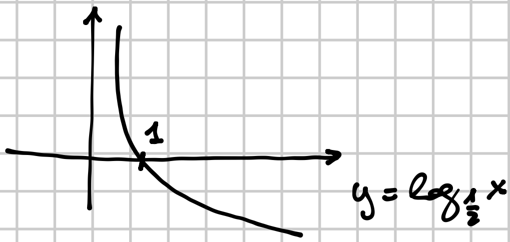
$$\frac{\cancel{n}\left(1-\frac{1}{n}\right)}{\cancel{n}\left(1+\frac{2}{n}\right)}\to 1$$

$$\lim_{n \to +\infty} \ln(\underbrace{n^2 + n}_{+\infty}) = \ln(+\infty) = +\infty$$



Se fosse

$$\lim_{n \to +\infty} \log_{\frac{1}{2}} (n^2+n) = \log_{\frac{1}{2}}(+\infty) = -\infty$$



$$\lim_{n \to +\infty} \ln\left(1 + \underbrace{\frac{2}{n+2}}_{0}\right) = \ln(1) = 0$$

$$\lim_{n \to +\infty} \left(\sqrt{n+1} - \sqrt{n-2}\right) = +\infty - \infty$$

$$= \lim_{n \to +\infty} \left(\sqrt{n+1} - \sqrt{n-2}\right) \cdot \frac{\sqrt{n+1} + \sqrt{n-2}}{\sqrt{n+1} + \sqrt{n-2}} =$$

$$= \lim_{n \to +\infty} \frac{\cancel{n} + 1 - \cancel{n} + 2}{\sqrt{n+1} + \sqrt{n-2}} = \frac{3}{+\infty} =$$

$$\lim_{n\to+\infty}\left(\sqrt{n^2+1}-\sqrt{n}\right) = +\infty-\infty$$

$$= \lim_{n\to+\infty}\left(\sqrt{n^2+1}-\sqrt{n}\right)\cdot\frac{\sqrt{n^2+1}+\sqrt{n}}{\sqrt{n^2+1}+\sqrt{n}} =$$

$$= \lim_{n\to+\infty}\frac{n^2+1-n}{\sqrt{n^2+1}+\sqrt{n}} = \frac{+\infty}{+\infty} \quad \text{F.I.}$$

$$= \lim_{n\to+\infty}\frac{n^2\left(1+\frac{1}{n^2}-\frac{1}{n}\right)}{\sqrt{n^2\left(1+\frac{1}{n^2}\right)}+\sqrt{n}} = \lim_{n\to+\infty}\frac{n^2\left(1+\frac{1}{n^2}-\frac{1}{n}\right)}{n\sqrt{1+\frac{1}{n^2}}+\sqrt{n}} =$$

$$= \lim_{n\to+\infty}\frac{n^2\left(1+\frac{1}{n^2}-\frac{1}{n}\right)}{n\left(\sqrt{1+\frac{1}{n^2}}+\frac{\sqrt{n}}{n}\right)} = \lim_{n\to+\infty}\frac{n^{\cancel{2}}\left(1+\underset{\to 0}{\frac{1}{n^2}}-\underset{\to 0}{\frac{1}{n}}\right)}{\cancel{n}\left(\sqrt{1+\underset{\downarrow 0}{\frac{1}{n^2}}}+\underset{\downarrow 0}{\frac{1}{\sqrt{n}}}\right)} = \frac{+\infty\cdot(1+0-0)}{\sqrt{1+0}+0} =$$

$$= \frac{+\infty}{1} = +\infty$$

Si poteva anche risolvere così:

$$\lim_{n\to+\infty}\left(\sqrt{n^2+1}-\sqrt{n}\right) = \lim_{n\to+\infty}\left(\sqrt{n^2\left(1+\frac{1}{n^2}\right)}-\sqrt{n}\right) =$$

$$= \lim_{n\to+\infty}\left(n\sqrt{1+\frac{1}{n^2}}-\sqrt{n}\right) = \lim_{n\to+\infty} n\left(\sqrt{1+\underset{\downarrow 0}{\frac{1}{n^2}}}-\underset{\downarrow 0}{\frac{1}{\sqrt{n}}}\right) = +\infty\cdot 1 = +\infty$$

$\lim_{n\to+\infty} \sin(n)$ NON ESISTE

$\lim_{n\to+\infty} \cos(n)$ NON ESISTE

$\lim_{n\to+\infty} \tan(n)$ NON ESISTE

ATTENZIONE $\lim_{n\to+\infty} \sin(n\pi) = \lim_{n\to+\infty} 0 = 0$

$$\lim_{n\to+\infty} \left[n + \sin(n)\right] = +\infty$$

Posso applicare il TEOREMA DEI CARABINIERI (variante)

$$\forall n \quad c_n \leq a_n \quad e \quad \lim_{n\to+\infty} c_n = +\infty \implies \exists \lim_{n\to+\infty} a_n = +\infty$$

$a_n = n + \sin(n)$

$c_n = n - 1$

$$\underbrace{n - 1}_{\to +\infty} \leq n + \sin(n)$$

quindi anche $n + \sin(n) \to +\infty$

$$\lim_{n\to+\infty} \sqrt{\frac{n+1}{9n}} = \lim_{n\to+\infty} \sqrt{\frac{\cancel{n}(1+\frac{1}{n})}{9\cancel{n}}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

$\frac{1}{n} \to 0$

$$\lim_{n\to+\infty} \log_{\frac{1}{3}} \frac{9n}{9+n} = \lim_{n\to+\infty} \log_{\frac{1}{3}} \frac{9\cancel{n}}{\cancel{n}\left(\frac{9}{n}+1\right)} = \log_{\frac{1}{3}} 9 = -2$$

$\frac{9}{n} \to 0$

$$\lim_{n\to+\infty} \cos\left(\frac{3}{3n^3+1}\right) = \cos(0) = 1$$

$\frac{3}{3n^3+1} \to 0$