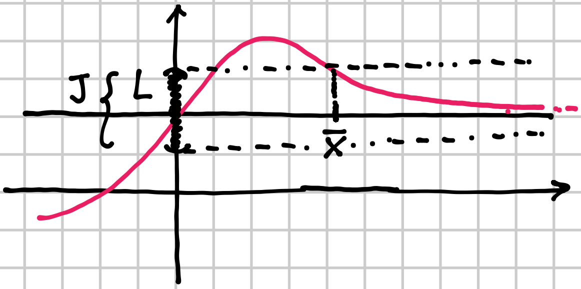
DAL LIMITE DI SUCCESSIONI AL LIMITE DI FUNZIONI

$\lim_{n\to+\infty} a_n = L \in \overline{\mathbb{R}}$ $\iff$ $\forall J$ intorno di L $\exists m : \forall n \geq m$ $a_n \in J$

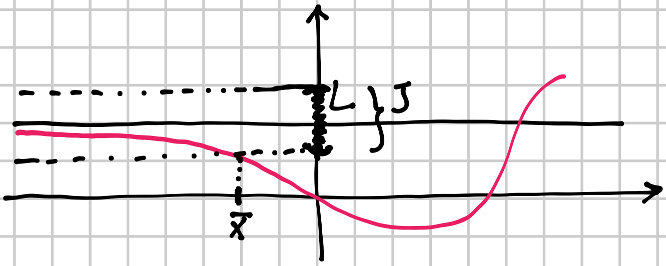
Se al posto di una successione ho una funzione reale $f : \mathbb{R} \to \mathbb{R}$, come posso trasferire il concetto di limite (per $x \to +\infty$)

$\lim_{x\to+\infty} f(x) = L$



$\forall J$ intorno di L $\exists \bar{x} : \forall x \geq \bar{x}$ $f(x) \in J$

$\lim_{x\to-\infty} f(x) = L$



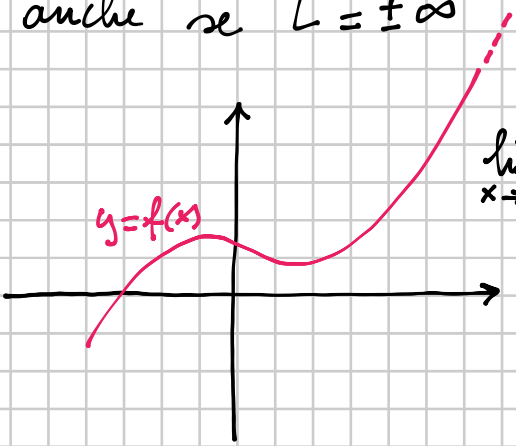
$\forall J$ intorno di L $\exists \bar{x} : \forall x \leq \bar{x}$ $f(x) \in J$

Sia $x_0 = \pm\infty$, cioè $x_0 \in \{+\infty, -\infty\}$. Posso unificare le 2 condizioni precedenti

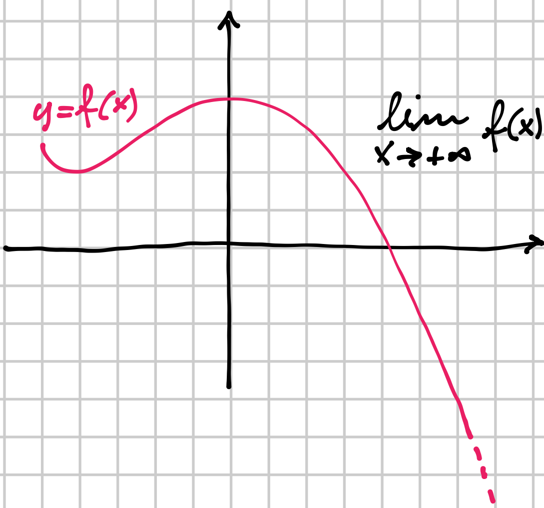
$$\boxed{\forall J \text{ intorno di } L \quad \exists I \text{ intorno di } x_0 : \quad \forall x \in I \quad f(x) \in J}$$

x_0 è $+\infty$ oppure $-\infty$

Vale anche se $L = \pm\infty$



$\lim_{x\to+\infty} f(x) = +\infty$



$\lim_{x\to+\infty} f(x) = -\infty$

135 $\lim_{x \to -\infty} \frac{9x^3 + x^2}{2}$ $[-\infty]$

$$\lim_{x \to -\infty} \frac{9x^3+x^2}{2} = \frac{9(-\infty)^3+(-\infty)^2}{2} = \frac{-\infty+\infty}{2} \text{ F.I.}$$

$$= \lim_{x \to -\infty} \frac{\overset{-\infty}{\overset{\uparrow}{x^3}}\left(9+\overset{0}{\overset{\uparrow}{\frac{1}{x}}}\right)}{2} = \frac{-\infty \cdot 9}{2} = \boxed{-\infty}$$

141 $\lim_{x \to -\infty} (\sqrt{1-2x} - \sqrt{3-2x}) = +\infty - \infty$ F.I. $[0]$

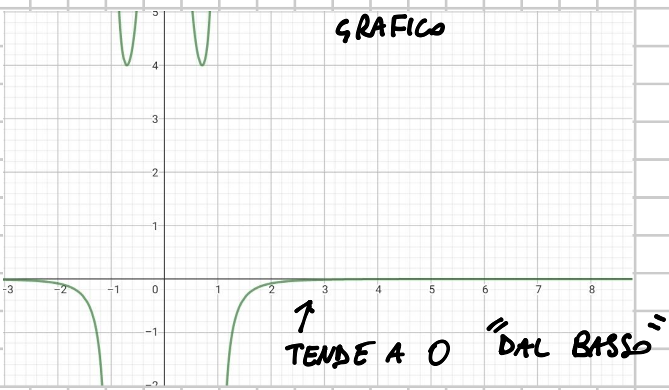
$$= \lim_{x \to -\infty} \left(\sqrt{1-2x} - \sqrt{3-2x}\right) \cdot \frac{\sqrt{1-2x}+\sqrt{3-2x}}{\sqrt{1-2x}+\sqrt{3-2x}} =$$

$$= \lim_{x \to -\infty} \frac{1-\cancel{2x}-3+\cancel{2x}}{\sqrt{1-2x}+\sqrt{3-2x}} = \lim_{x \to -\infty} \frac{-2}{\sqrt{1-2x}+\sqrt{3-2x}} =$$

$$= \frac{-2}{+\infty} = 0$$

148 $\lim_{x \to +\infty} \frac{1}{-x^4 + x^2} = \frac{1}{-\infty+\infty}$ F.I. $[0^-]$

$$= \lim_{x \to +\infty} \frac{1}{\underbrace{x^4}_{+\infty}\left(-1+\underbrace{\frac{1}{x^2}}_{0}\right)} = \frac{1}{+\infty(-1)} = \frac{1}{-\infty} = 0^-$$



152 $\lim\limits_{x \to -\infty} 4^{x^3+x^2} = \lim\limits_{x \to -\infty} 4^{-\infty+\infty}$ F.I. [0]

$$= \lim_{x \to -\infty} 4^{\overbrace{x^3}^{-\infty}\left(1+\underbrace{\frac{1}{x}}_{0}\right)} = 4^{-\infty} = 0^+$$