147 **EUREKA!** $\lim_{x\to+\infty}\left(\sqrt[4]{x^3}-\sqrt[3]{x^2}+\sqrt{x}-x\right) =$ $[-\infty]$

$= +\infty - \infty + \infty - \infty$ F.I.

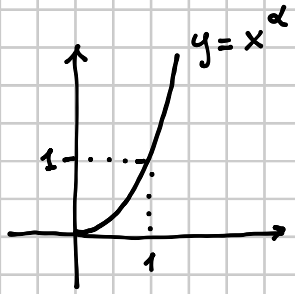
$$= \lim_{x\to+\infty}\left(x^{\frac{3}{4}} - x^{\frac{2}{3}} + x^{\frac{1}{2}} - x\right) = \lim_{x\to+\infty} x\left(x^{\frac{3}{4}-1} - x^{\frac{2}{3}-1} + x^{\frac{1}{2}-1} - 1\right) =$$

$$= \lim_{x\to+\infty} x\left(x^{-\frac{1}{4}} - x^{-\frac{1}{3}} + x^{-\frac{1}{2}} - 1\right) =$$

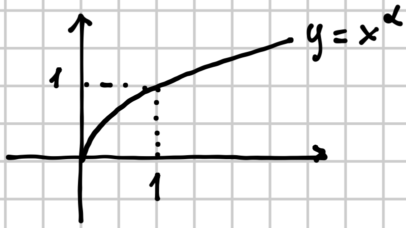
$$= \lim_{x\to+\infty} \underset{\substack{\uparrow\\ +\infty}}{x}\left(\underbrace{\frac{1}{\sqrt[4]{x}}}_{\substack{\downarrow\\ 0}} - \underbrace{\frac{1}{\sqrt[3]{x}}}_{\substack{\downarrow\\ 0}} + \underbrace{\frac{1}{\sqrt{x}}}_{\substack{\downarrow\\ 0}} - 1\right) = +\infty(-1) = -\infty$$

x^{α} $\quad \alpha > 0$ reale

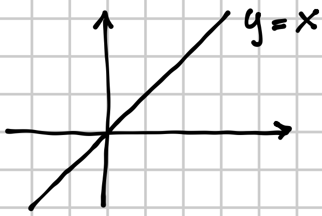
$\alpha > 1$



$0 < \alpha < 1$

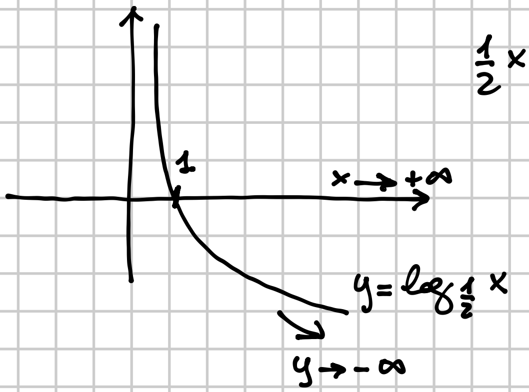


$\alpha = 1$



151 $\lim_{x\to-\infty}\log_{\frac{1}{2}}\underbrace{\left(\frac{1}{2}x^2-x\right)}_{\downarrow\ +\infty} = \log_{\frac{1}{2}}(+\infty) = -\infty$

$$\frac{1}{2}x^2-x=\underbrace{x^2}_{\downarrow\ +\infty}\left(\frac{1}{2}-\underbrace{\frac{1}{x}}_{\downarrow\ 0}\right)\longrightarrow +\infty\cdot\frac{1}{2}=+\infty$$

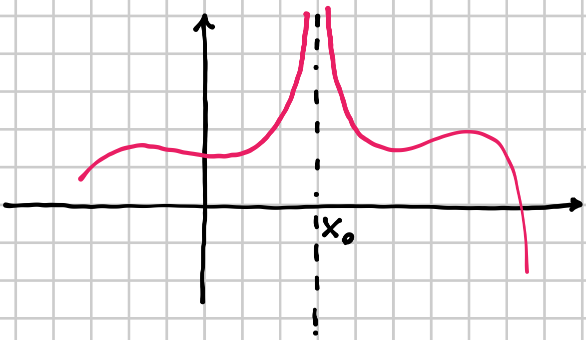


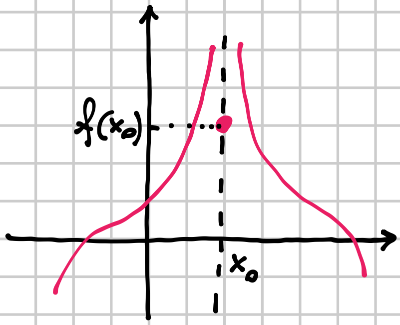
LIMITE DI $f(x)$ PER $X\to X_0$ FINITO

x_0 punto di accumulazione per il dominio della funzione, x_0 FINITO

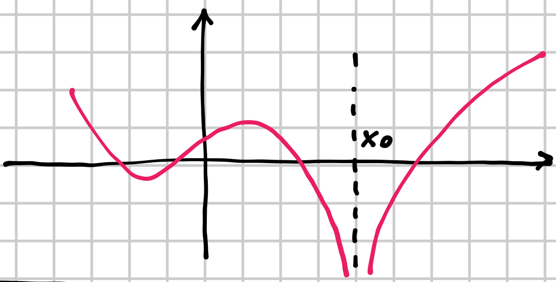
L infinito

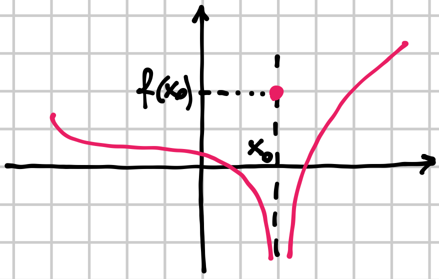
$\lim_{x\to x_0} f(x)=+\infty$





$\lim_{x\to x_0} f(x)=-\infty$



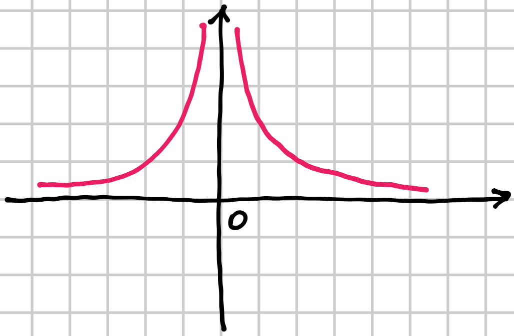


$\boxed{\text{In } x_0 \text{ la funzione può essere definita oppure no}}$

x_0 può anche non essere nel dominio. L'importante è che sia un punto di accumulazione per il dominio.

ESEMPI

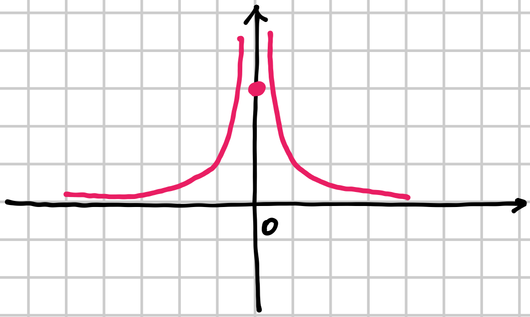
1) $f: \mathbb{R} \setminus \{0\} \to \mathbb{R}$ $\qquad f(x) = \dfrac{1}{|x|}$



$x_0 = 0$ non è nel dominio di f, ma è comunque un punto di accumulazione per il dominio

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1}{|x|} = +\infty$$

2) $f: \mathbb{R} \to \mathbb{R}$ $\qquad f(x) = \begin{cases} \dfrac{1}{|x|} & \text{se } x \neq 0 \\ 3 & \text{se } x = 0 \end{cases}$

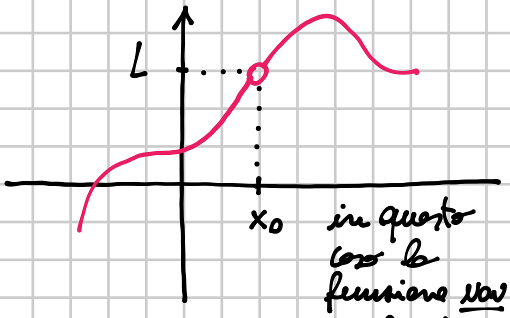


$$\lim_{x \to 0} f(x) = +\infty \qquad (\text{anche se } f(0) = 0)$$

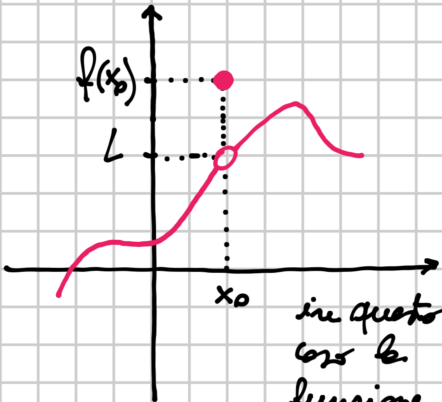
Vediamo qual è la situazione se L è finito

$$\lim_{x \to x_0} f(x) = L$$

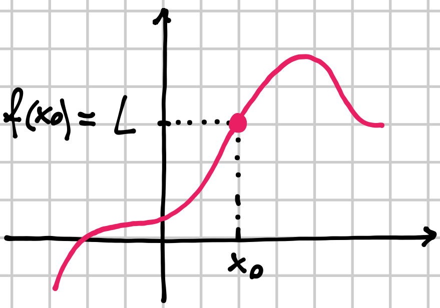
x_0 finito, L finito



in questo caso la funzione NON è definita in x_0



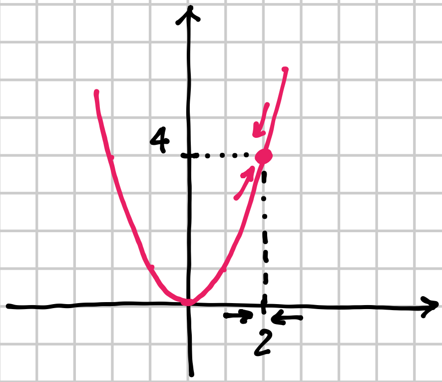
in questo caso la funzione È definita in x_0, ma in x_0 vale $f(x_0) \neq L$



in questo caso la funzione È definita in x_0 e il limite L coincide con $f(x)$ (f è CONTINUA in x_0)

ESEMPI

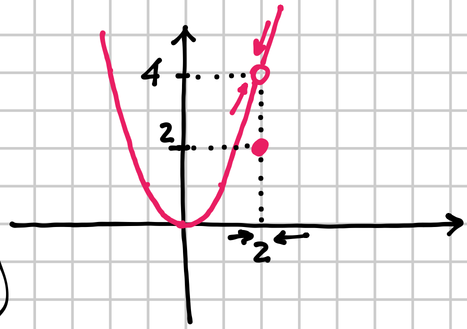
1) $\lim_{x \to 2} x^2 = 2^2 = 4$



$f: \mathbb{R} \to \mathbb{R}$ $\quad f(x) = x^2$

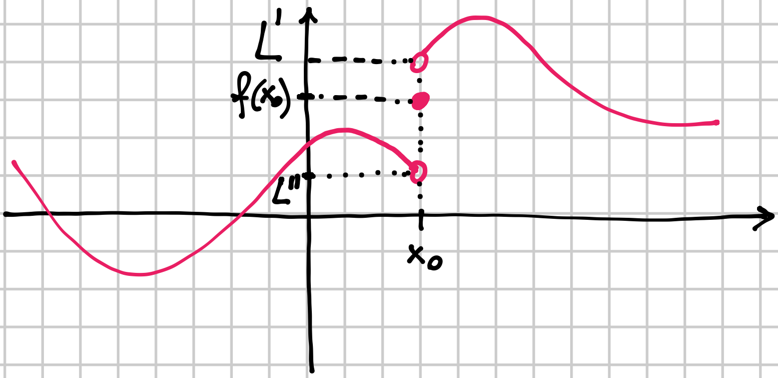
è continua in 2

2) $f: \mathbb{R} \to \mathbb{R}$ $\quad f(x) = \begin{cases} x^2 & \text{se } x \neq 2 \\ 2 & \text{se } x = 2 \end{cases}$



$\lim_{x \to 2} f(x) = 4$ $\quad$ (anche se $f(2) = 2$)

LIMITI DESTRO E SINISTRO



$\lim_{x \to x_0^+} f(x) = L'$

$\lim_{x \to x_0^-} f(x) = L''$

$\lim_{x \to x_0} f(x)$ NON ESISTE

Come al solito, quello che succede in x_0 non mi interessa.

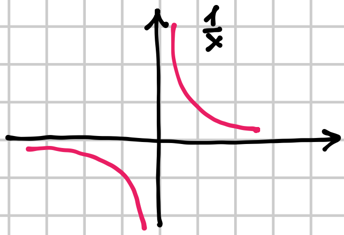
ESEMPIO

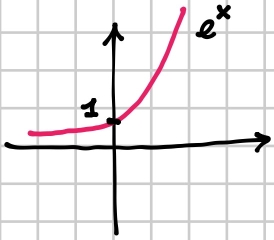
$f(x) = e^{\frac{1}{x}} \qquad f: \mathbb{R} \setminus \{0\} \to \mathbb{R}$

$\longrightarrow D = (-\infty, 0) \cup (0, +\infty)$

$\lim_{x \to -\infty} e^{\frac{1}{x}} = e^{0^-} = 1^-$

$\lim_{x \to 0^-} e^{\frac{1}{x}} = e^{-\infty} = 0^+$

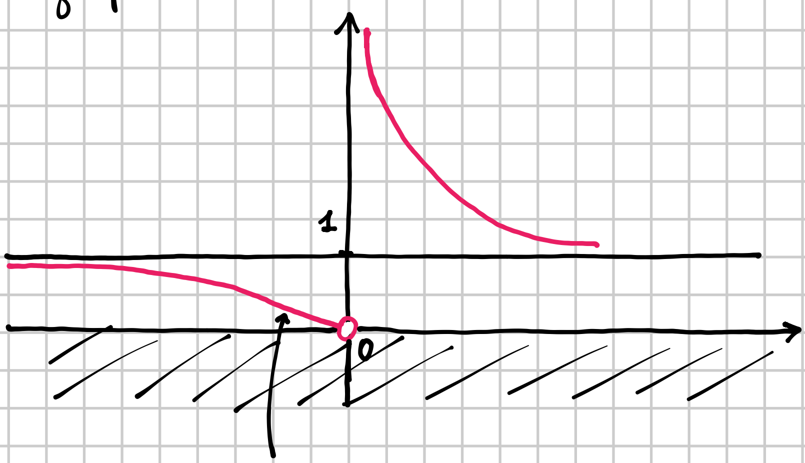




$\lim_{x \to 0^+} e^{\frac{1}{x}} = e^{+\infty} = +\infty$

$\lim_{x \to +\infty} e^{\frac{1}{x}} = e^{0^+} = 1^+$

Come è fatto questo grafico?



per il momento non possiamo prevedere altro qui