206 $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{9x^4+5x^2}}{(x+2)^2} =$ [3]

$$= \lim_{x\to-\infty}\frac{\sqrt{x^4\left(9+\frac{5}{x^2}\right)}}{x^2+4x+4} = \lim_{x\to-\infty}\frac{\cancel{x^2}\sqrt{9+\underbrace{\frac{5}{x^2}}_{\to 0}}}{\cancel{x^2}\left(1+\underbrace{\frac{4}{x}}_{0}+\underbrace{\frac{4}{x^2}}_{0}\right)} = \frac{3}{1} = \boxed{3}$$

214 $\displaystyle\lim_{x\to-2}\frac{2x^2+3x-2}{x^2+2x} = \frac{2(-2)^2+3(-2)-2}{(-2)^2+2(-2)} = \frac{0}{0}$ F.I.

$$= \lim_{x\to-2}\frac{(2x-1)\cancel{(x+2)}}{x\cancel{(x+2)}} = \frac{2(-2)-1}{-2} = \frac{-5}{-2} = \boxed{\frac{5}{2}}$$

$$\begin{array}{r|rr|r} & 2 & 3 & -2 \\ -2 & & -4 & 2 \\ \hline & 2 & -1 & // \end{array}$$

229 $\displaystyle\lim_{x\to2^+}\left[\frac{x^3-2x^2}{\underbrace{x^2-4x+4}_{(x-2)^2}}+\frac{2x^2-8}{(x-2)^2}\right] =$ [$+\infty$]

$$= \lim_{x\to2^+}\frac{x^3-\cancel{2x^2}+\cancel{2x^2}-8}{(x-2)^2} = \lim_{x\to2^+}\frac{x^3-8}{(x-2)^2} = \frac{8-8}{(2-2)^2} = \frac{0}{0} \text{ F.I.}$$

$$= \lim_{x\to2^+}\frac{\cancel{(x-2)}(x^2+2x+4)}{(x-2)^{\cancel{2}}} = \frac{4+4+4}{0^+} = \frac{12}{0^+} = \boxed{+\infty}$$

243 $\lim_{x\to 2} \frac{\sqrt{x^2-2x+9}-3}{x-2} = \frac{0}{0}$ F.I. $\left[\frac{1}{3}\right]$

$$= \lim_{x\to 2} \frac{\sqrt{x^2-2x+9}-3}{x-2} \cdot \frac{\sqrt{x^2-2x+9}+3}{\sqrt{x^2-2x+9}+3} =$$

$$= \lim_{x\to 2} \frac{x^2-2x\cancel{+9}\cancel{-9}}{(x-2)\left(\sqrt{x^2-2x+9}+3\right)} = \lim_{x\to 2} \frac{x\cancel{(x-2)}}{\cancel{(x-2)}\left(\sqrt{x^2-2x+9}+3\right)} =$$

$$= \frac{2}{6} = \boxed{\frac{1}{3}}$$

310 $\lim_{x\to+\infty} \left(\frac{3x-2}{x+1}\right)^{\frac{x-1}{2x}} = 3^{\frac{1}{2}} = \sqrt{3}$ $[\sqrt{3}]$

$$\lim_{x\to+\infty} \frac{3x-2}{x+1} = \frac{\cancel{x}\left(3-\frac{2}{x}\right)}{\cancel{x}\left(1+\frac{1}{x}\right)} = 3$$

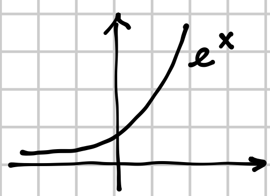
$$\lim_{x\to+\infty} \frac{x-1}{2x} = \lim_{x\to+\infty} \frac{\cancel{x}\left(1-\frac{1}{x}\right)}{2\cancel{x}} = \frac{1}{2}$$

304 $\lim_{x\to-\infty} (2x^2+x)^x =$ $[0^+]$

$$= \lim_{x\to-\infty} e^{\ln(2x^2+x)^x} = \lim_{x\to-\infty} e^{x\ln(\overbrace{2x^2+x}^{+\infty})} = e^{-\infty\cdot(+\infty)} =$$

$$\lim_{x\to-\infty} (2x^2+x) = \lim_{x\to-\infty} \overset{+\infty}{x^2}\Big(2+\underset{0}{\frac{1}{x}}\Big) = +\infty \cdot 2 = +\infty$$

$$= e^{-\infty} = 0^+$$



323 $\lim_{x\to -1} \log_9 \frac{\sqrt[3]{x}+1}{x+1}$ $\left[-\frac{1}{2}\right]$

Prima di tutto calcolo:

$$\lim_{x\to -1} \frac{\sqrt[3]{x}+1}{x+1} = \frac{0}{0} \quad \text{F.I.}$$

$(a+b)(a^2-ab+b^2)=a^3+b^3$

FALSO QUADRATO

$$= \lim_{x\to -1} \frac{\sqrt[3]{x}+1}{x+1} \cdot \frac{(\sqrt[3]{x})^2-\sqrt[3]{x}+1}{(\sqrt[3]{x})^2-\sqrt[3]{x}+1} =$$

$$= \lim_{x\to -1} \frac{\cancel{x+1}}{\cancel{(x+1)}(\sqrt[3]{x^2}-\sqrt[3]{x}+1)} = \frac{1}{1+1+1} = \frac{1}{3}$$

$$\lim_{x\to -1} \log_9 \frac{\sqrt[3]{x}+1}{x+1} = \log_9 \frac{1}{3} = -\frac{1}{2}$$

(esponente da dare a 9 per ottenere $\frac{1}{3}$)

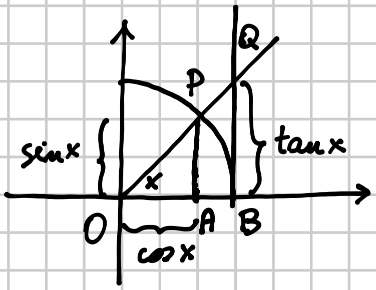
LIMITI NOTEVOLI

Prendiamo come definizione $\boxed{e = \lim_{x \to \pm\infty} \left(1 + \frac{1}{x}\right)^x}$

1) $\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$

DIMOSTRAZIONE

$x > 0$



Area triangolo OAP < Area settore circolare OBP < Area triangolo OBQ

$$\cancel{\frac{1}{2}} \sin x \cdot \cos x < \cancel{\frac{1}{2}} x < \cancel{\frac{1}{2}} \tan x$$

$$\sin x \cdot \cos x < x < \frac{\sin x}{\cos x}$$

divido per $\sin x$

$$\cos x < \frac{x}{\sin x} < \frac{1}{\cos x}$$

prendo i reciproci

$$\cos x < \frac{\sin x}{x} < \frac{1}{\cos x}$$

per $x \to 0$: $\cos x \to 1$, $\frac{1}{\cos x} \to 1$

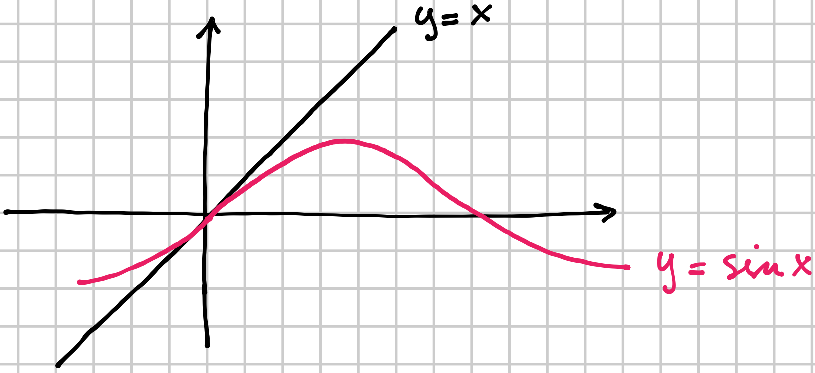
$\frac{\sin x}{x} \to 1$ per il teorema dei 2 carabinieri

Se $x < 0$

$$\lim_{x \to 0^-} \frac{\sin x}{x} = \lim_{t \to 0^+} \frac{\sin(-t)}{-t} = \lim_{t \to 0^+} \frac{-\sin t}{-t} = 1$$

pongo $x = -t$

$t = -x$



2) $$\boxed{\lim_{x\to 0} \frac{1-\cos x}{x} = 0}$$

DIMOSTRAZIONE

$$\lim_{x\to 0} \frac{1-\cos x}{x}\cdot\frac{1+\cos x}{1+\cos x} = \lim_{x\to 0} \frac{1-\cos^2 x}{x(1+\cos x)} =$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x(1+\cos x)} = \lim_{x\to 0} \underbrace{\frac{\sin x}{x}}_{\downarrow 1} \cdot \frac{\overset{\to 0}{\sin x}}{1+\underbrace{\cos x}_{\downarrow 1}} = 0$$

3) $$\boxed{\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}}$$

DIMOSTRAZIONE

$$\lim_{x\to 0} \frac{1-\cos x}{x^2}\cdot\frac{1+\cos x}{1+\cos x} = \lim_{x\to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)} = \lim_{x\to 0} \frac{\sin^2 x}{x^2(1+\cos x)} =$$

$$= \lim_{x\to 0} \underbrace{\frac{\sin x}{x}}_{\downarrow 1} \cdot \underbrace{\frac{\sin x}{x}}_{\downarrow 1} \cdot \frac{1}{1+\underbrace{\cos x}_{\downarrow 1}} = \frac{1}{2}$$

4) $$\boxed{\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1}$$

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = \lim_{x\to 0} \frac{1}{x}\ln(1+x) = \lim_{x\to 0} \ln(1+x)^{\frac{1}{x}} =$$

$t = \frac{1}{x}$

$t \to \infty$

se

$x \to 0$

$$= \lim_{t\to\infty} \ln \underbrace{\left(1+\frac{1}{t}\right)^t}_{\downarrow e} = \ln(e) = 1$$

5) $$\boxed{\lim_{x \to 0} \frac{e^x - 1}{x} = 1}$$

DIMOSTRAZIONE

$$\lim_{x \to 0} \frac{e^x - 1}{x} = \lim_{t \to 0} \frac{t}{\ln(t+1)} = \lim_{t \to 0} \frac{1}{\underbrace{\frac{\ln(t+1)}{t}}_{\to 1}} = 1$$

$t = e^x - 1 \qquad t \to 0$ per $x \to 0$

$e^x = t + 1$

$x = \ln(t+1)$

6) $\alpha \in \mathbb{R}$

$$\boxed{\lim_{x \to 0} \frac{(1+x)^\alpha - 1}{x} = \alpha}$$

DIMOSTRAZIONE

$$\lim_{x \to 0} \frac{(1+x)^\alpha - 1}{x} = \lim_{x \to 0} \frac{e^{\ln(1+x)^\alpha} - 1}{x} =$$

$$= \lim_{x \to 0} \frac{e^{\alpha \ln(1+x)} - 1}{x} \cdot \frac{\alpha \ln(1+x)}{\alpha \ln(1+x)} =$$

$t = \alpha \ln(1+x)$

$t \to 0$ per $x \to 0$

$$= \lim_{x \to 0} \frac{e^{\alpha \ln(1+x)} - 1}{\alpha \ln(1+x)} \cdot \frac{\alpha \ln(1+x)}{x} =$$

$$= \lim_{x \to 0} \frac{e^{\alpha \ln(1+x)} - 1}{\alpha \ln(1+x)} \cdot \lim_{x \to 0} \frac{\alpha \ln(1+x)}{x} =$$

$$= \lim_{t \to 0} \underbrace{\frac{e^t - 1}{t}}_{\to 1} \cdot \lim_{x \to 0} \alpha \underbrace{\frac{\ln(1+x)}{x}}_{\to 1} =$$

$$= 1 \cdot \alpha = \alpha$$

356 $\lim_{x\to 0} \frac{x^2+x}{2x+\sin x} = \frac{0}{0}$ F.I.

$$= \lim_{x\to 0} \frac{\cancel{x}(x+1)}{\cancel{x}\left(2+\underbrace{\frac{\sin x}{x}}_{1}\right)} = \frac{1}{2+1} = \frac{1}{3}$$

374 $\lim_{x\to 0} \frac{\sin 5x}{x} = \frac{0}{0}$ [5]

$$\lim_{x\to x_0} \frac{\sin f(x)}{f(x)} = 1$$

se $f(x)\to 0$ per $x\to x_0$

$$= \lim_{x\to 0} \underbrace{\frac{\sin 5x}{5x}}_{1} \cdot 5 = 5$$

412 $\lim_{x\to +\infty} \left(1+\frac{1}{4x}\right)^x = \lim_{x\to +\infty} \left\{\left[\left(1+\frac{1}{4x}\right)^x\right]^4\right\}^{\frac{1}{4}} =$

$$= \lim_{x\to +\infty} \left[\underbrace{\left(1+\frac{1}{4x}\right)^{4x}}_{e}\right]^{\frac{1}{4}} = e^{\frac{1}{4}} = \sqrt[4]{e}$$

414 $\lim_{x\to 0} \frac{(1+4x^2)^3-1}{x^2} =$

$$= \lim_{x\to 0} \underbrace{\frac{(1+4x^2)^3-1}{4x^2}}_{3} \cdot 4 = 3\cdot 4 = 12$$

428

$$\lim_{x\to 0}\frac{e^{3x}-e^{2x}}{x^2-3x}=\frac{0}{0}\quad \text{F.I.}$$

$$=\lim_{x\to 0}\frac{e^{2x}\left(e^x-1\right)}{x\left(x-3\right)}\rightarrow 1 \;=\frac{1\cdot 1}{-3}=\boxed{-\frac{1}{3}}$$