386 $\lim_{x \to 2} \frac{1-\sqrt{\cos(x-2)}}{x^2-4x+4} = \left[\frac{1}{4}\right]$

$= \frac{0}{0}$ F.I.

$= \lim_{x \to 2} \frac{1-\sqrt{\cos(x-2)}}{(x-2)^2} = \lim_{t \to 0} \frac{1-\sqrt{\cos t}}{t^2} \cdot \frac{1+\sqrt{\cos t}}{1+\sqrt{\cos t}} =$

$t = x-2$

$x \to 2 \Rightarrow t \to 0$

$= \lim_{t \to 0} \frac{\overbrace{1-\cos t}^{\to \frac{1}{2}}}{t^2 \underbrace{(1+\sqrt{\cos t})}_{\to 2}} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

410 $\lim_{x \to 0} \frac{\ln(x+5) - \ln 5}{x} = \frac{0}{0}$ F.I.

$= \lim_{x \to 0} \frac{\ln\left(\frac{x+5}{5}\right)}{x} = \lim_{x \to 0} \frac{\ln\left(\frac{x}{5}+1\right)}{x} = \lim_{t \to 0} \frac{\overbrace{\ln(t+1)}^{\to 1}}{5t} = \frac{1}{5}$

$t = \frac{x}{5} \Rightarrow x = 5t$

$x \to 0 \Rightarrow t \to 0$

427 $\lim_{x \to 4} \frac{\ln(x-3)}{x-4} = \frac{0}{0}$ F.I.

LIMITE NOTEVOLE

$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$

$= \lim_{x \to 4} \frac{\ln(1+x-4)}{x-4} = \lim_{t \to 0} \frac{\ln(1+t)}{t} = 1$

$t = x-4$

$x \to 4 \Rightarrow t \to 0$

447 $\lim_{x\to 0} \frac{\sqrt[6]{1-x}-1}{e^{2x}-1} = \frac{0}{0}$

$$= \lim_{x\to 0} \frac{\sqrt[6]{1-x}-1}{e^{2x}-1} \cdot \frac{x}{x} =$$

$$= \lim_{x\to 0} \frac{(1+(-x))^{\frac{1}{6}}-1}{x} \cdot \frac{x}{e^{2x}-1} \cdot \frac{-1}{-1} \cdot \frac{2}{2} =$$

$$= \lim_{x\to 0} \underbrace{\frac{(1+(-x))^{1/6}-1}{-x}}_{\to \frac{1}{6}} \cdot \underbrace{\frac{2x}{e^{2x}-1}}_{\to 1} \cdot \left(-\frac{1}{2}\right) = \frac{1}{6} \cdot 1 \cdot \left(-\frac{1}{2}\right) = -\frac{1}{12}$$

442 $\lim_{x\to 0} \frac{e^{2+x^2}-e^2}{1-\cos^2 x} = \frac{0}{0}$ F.I.

$$= \lim_{x\to 0} \frac{e^2(e^{x^2}-1)}{1-\cos^2 x} = \lim_{x\to 0} \frac{e^2(e^{x^2}-1)}{\sin^2 x} \cdot \frac{x^2}{x^2} =$$

$$= \lim_{x\to 0} \frac{e^2(e^{x^2}-1)}{x^2} \cdot \frac{x^2}{\sin^2 x} = \lim_{x\to 0} e^2 \underbrace{\frac{(e^{x^2}-1)}{x^2}}_{\to 1} \cdot \Big(\underbrace{\frac{x}{\sin x}}_{\to 1}\Big)^2 = e^2$$

430 $\lim_{x\to\pm\infty}\left(\frac{3x-1}{3x+2}\right)^{\frac{x}{2}} = 1^{\infty}$ F.I. $\left[\frac{1}{\sqrt{e}}\right]$

$$= \lim_{x\to\infty}\left(\frac{3x+2-2-1}{3x+2}\right)^{\frac{x}{2}} =$$

$$= \lim_{x\to\infty}\left(\frac{3x+2}{3x+2} - \frac{3}{3x+2}\right)^{\frac{x}{2}} = \lim_{x\to\infty}\left(1 - \frac{1}{x+\frac{2}{3}}\right)^{\frac{x}{2}} =$$

$$= \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{-\frac{t}{2}-\frac{1}{3}} =$$

$$= \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{-\frac{t}{2}}\cdot\left(1+\frac{1}{t}\right)^{-\frac{1}{3}} =$$

$$= \lim_{t\to\infty}\underbrace{\left[\left(1+\frac{1}{t}\right)^{t}\right]^{-\frac{1}{2}}}_{\downarrow\, e^{-\frac{1}{2}}}\cdot\underbrace{\left(1+\overset{0}{\frac{1}{t}}\right)^{-\frac{1}{3}}}_{\downarrow\, 1} = e^{-\frac{1}{2}}\cdot 1 =$$

$$-\frac{1}{x+\frac{2}{3}} = \frac{1}{t}$$

$$\Downarrow$$

$$t = -x-\frac{2}{3} \Rightarrow x = -t-\frac{2}{3}$$

$$x\to+\infty \Rightarrow t\to-\infty$$

$$x\to-\infty \Rightarrow t\to+\infty$$

$$\frac{x}{2} = -\frac{t}{2}-\frac{1}{3}$$

$$= e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

PROVIAMO :

$$\lim_{x\to\infty}\left(\frac{3x-1}{3x+2}\right)^{\frac{x}{2}} = \lim_{x\to\infty} e^{\ln\left(\frac{3x-1}{3x+2}\right)^{\frac{x}{2}}} = \lim_{x\to\infty} e^{\overbrace{\frac{x}{2}\ln\left(\frac{3x-1}{3x+2}\right)}^{\infty\cdot 0 \text{ F.I.}}} = (*)$$

V. DOPO

<u>A PARTE</u> :

$$\lim_{x\to\infty}\frac{x}{2}\ln\left(\frac{3x-1}{3x+2}\right) = \lim_{x\to\infty}\frac{x}{2}\ln\left(1-\frac{1}{x+\frac{2}{3}}\right) = \qquad x = -t-\frac{2}{3}$$

come prima

$$= \lim_{t\to\infty}\frac{-t-\frac{2}{3}}{2}\ln\left(1+\frac{1}{t}\right) = \lim_{t\to\infty}\left[\frac{-t}{2}\ln\left(1+\frac{1}{t}\right)+\left(-\frac{1}{3}\right)\ln\left(1+\frac{1}{t}\right)\right] =$$

$$= \lim_{t \to \infty} \left[\frac{-t}{2} \ln\left(1+\frac{1}{t}\right) + \left(-\frac{1}{3}\right) \ln\left(1+\frac{1}{t}\right) \right] =$$

$$= \lim_{t \to \infty} \left[\underbrace{\frac{\ln\left(1+\frac{1}{t}\right)}{\frac{1}{t}}}_{\to 1} \left(-\frac{1}{2}\right) + \left(-\frac{1}{3}\right) \underbrace{\ln\left(1+\overset{\to 0}{\frac{1}{t}}\right)}_{\to 0} \right] = -\frac{1}{2}$$

ritornando al limite di potenza:

$$(*) = \lim_{x \to \infty} e^{\frac{x}{2} \ln\left(\frac{3x-1}{3x+2}\right)} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

FORME INDETERMINATE

$\frac{0}{0} \quad \frac{\infty}{\infty} \quad \infty \cdot 0 \quad 0 \cdot \infty \quad +\infty - \infty \quad -\infty + \infty$

ALTRE FORME INDETERMINATE

$0^0 \quad 1^\infty \quad \infty^0$

RICONDUCIBILI ALLE PRIME CON LA FORMULA

$$[f(x)]^{g(x)} = e^{g(x) \ln f(x)}$$

TENERE PRESENTE CHE VALGONO

$$\lim_{x \to 0} \frac{\arcsin x}{x} = 1$$

$\downarrow$

$x = \sin t$

$t = \arcsin x \Rightarrow t \to 0$ per $x \to 0$

$$\lim_{t \to 0} \frac{t}{\sin t} = 1$$

$$\lim_{x \to 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \to 0} \frac{\overset{\to 1}{\sin x}}{x \underbrace{\cos x}_{\to 1}} = 1$$

$$\lim_{x \to 0} \frac{\arctan x}{x} = 1$$

$\downarrow$

$x = \tan t$

$t = \arctan x \Rightarrow t \to 0$ per $x \to 0$

$$\lim_{t \to 0} \frac{t}{\tan t} = 1$$