444 $\lim_{x\to 0} \frac{\tan x}{e^{\sin x} - \cos x} = \frac{0}{0}$ F.I.

$$= \lim_{x\to 0} \frac{\sin x}{e^{\sin x} - 1 + 1 - \cos x} \cdot \frac{1}{\cos x} =$$

$$= \lim_{x\to 0} \frac{1}{\frac{e^{\sin x}-1}{\sin x} + \frac{1-\cos x}{\sin x}} \cdot \frac{1}{\cos x} =$$

$$= \lim_{x\to 0} \frac{1}{\underbrace{\frac{e^{\sin x}-1}{\sin x}}_{\to 1} + \underbrace{\frac{1-\cos x}{x}}_{\to 0} \cdot \underbrace{\frac{x}{\sin x}}_{\to 1}} \cdot \underbrace{\frac{1}{\cos x}}_{\to 1} = \frac{1}{1+1\cdot 0} \cdot \frac{1}{1} = 1$$

446 $\lim_{x\to 0} \frac{\ln(x+1)}{\sin 2x + \sin x} = \frac{0}{0}$ F.I.

$$= \lim_{x\to 0} \frac{\ln(x+1)}{\sin 2x + \sin x} \cdot \frac{x}{x} =$$

$$= \lim_{x\to 0} \frac{\ln(x+1)}{x} \cdot \frac{x}{\sin 2x + \sin x} =$$

$$= \lim_{x\to 0} \frac{\ln(x+1)}{x} \cdot \frac{1}{\frac{\sin 2x}{x} + \frac{\sin x}{x}} = \lim_{x\to 0} \underbrace{\frac{\ln(x+1)}{x}}_{\to 1} \cdot \frac{1}{\underbrace{\frac{\sin 2x}{2x}}_{\to 1} \cdot 2 + \underbrace{\frac{\sin x}{x}}_{\to 1}} =$$

$$= 1 \cdot \frac{1}{1\cdot 2 + 1} = \frac{1}{3}$$

539 $\displaystyle\lim_{x\to 0}\frac{\sqrt[4]{1+x^3}-1}{x^3-x^4} =$ $\left[\frac{1}{4}\right]$

$$= \lim_{x\to 0} \frac{(1+x^3)^{\frac{1}{4}}-1}{x^3(1-x)} = \frac{1}{4}$$

$\to \frac{1}{4}$, $\to 0$

APPLICO $\displaystyle\lim_{x\to 0}\frac{(1+x)^{\alpha}-1}{x}=\alpha$

526 $\displaystyle\lim_{x\to 0}(\cos x)^{-\frac{4}{x^2}} = 1^{\infty}$ F.I. $[e^2]$

$$= \lim_{x\to 0} e^{\ln(\cos x)^{-\frac{4}{x^2}}} = \lim_{x\to 0} e^{-\frac{4}{x^2}\ln(\cos x)} = \ldots (*)$$

Devo calcolare $\displaystyle\lim_{x\to 0} -\frac{4}{x^2}\ln(\cos x) =$

$1+t=\cos x$

$t=\cos x-1$

$x\to 0 \Rightarrow t\to 0$

$x=\arccos(1+t)$

$$= \lim_{t\to 0} -\frac{4}{(\arccos(1+t))^2}\cdot\frac{\ln(1+t)}{t}\cdot t = -4\cdot\frac{1}{2}\cdot 1 = -2$$

$\to \frac{1}{2}$, $\to 1$

Ci occupiamo di

$$\lim_{t\to 0}\frac{t}{(\arccos(1+t))^2} = \lim_{x\to 0}\frac{\cos x-1}{x^2} = -\frac{1}{2}$$

risostituisco $t=\cos x-1$

$\left(\text{perché } \displaystyle\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}\right)$

il limite finale è

$(*) = e^2$

507 $\lim_{x\to+\infty} \frac{\ln x}{\ln(x+2)} = \frac{\infty}{\infty}$ F.I. [1]

$$= \lim_{x\to+\infty} \frac{\ln(x+2-2)}{\ln(x+2)} = \lim_{x\to+\infty} \frac{\ln\left((x+2)\left(1-\frac{2}{x+2}\right)\right)}{\ln(x+2)} =$$

$$\boxed{\begin{array}{l} \ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b) \\ \ln(a\cdot b) = \ln(a) + \ln(b) \end{array}}$$

$$= \lim_{x\to+\infty} \frac{\ln(x+2) + \ln\left(1-\frac{2}{x+2}\right)}{\ln(x+2)} =$$

$$= \lim_{x\to+\infty} \left[1 + \frac{\overset{\to 0}{\ln\left(1-\frac{2}{x+2}\right)}}{\underset{\to +\infty}{\ln(x+2)}}\right] = 1$$

NON È UNA F.I.

$\frac{0}{\infty} = 0$ $\frac{\infty}{0} = \infty$

497 $\lim_{x\to 0} \frac{x+\tan x}{2x+\sin x} = \frac{0}{0}$ F.I. $\left[\frac{2}{3}\right]$

$$= \lim_{x\to 0} \frac{\cancel{x}\left(1+\overset{\to 1}{\frac{\tan x}{x}}\right)}{\cancel{x}\left(2+\underset{\to 1}{\frac{\sin x}{x}}\right)} = \frac{2}{3}$$