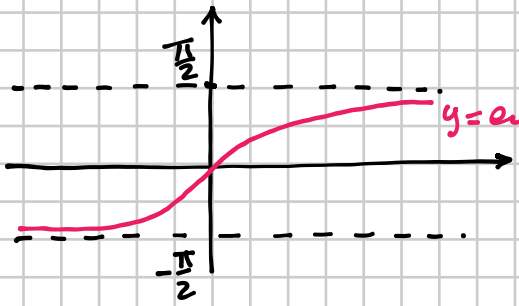
Trovare gli asintoti

1004 $y = \arctan(1 + x^2)$ $\left[y = \frac{\pi}{2}\right]$

$D = \mathbb{R} = (-\infty, +\infty)$ $\lim_{x \to \pm\infty} \arctan(1+x^2) = \frac{\pi}{2}$

$y = \frac{\pi}{2}$ è asintoto orizzontale per $x \to \pm\infty$

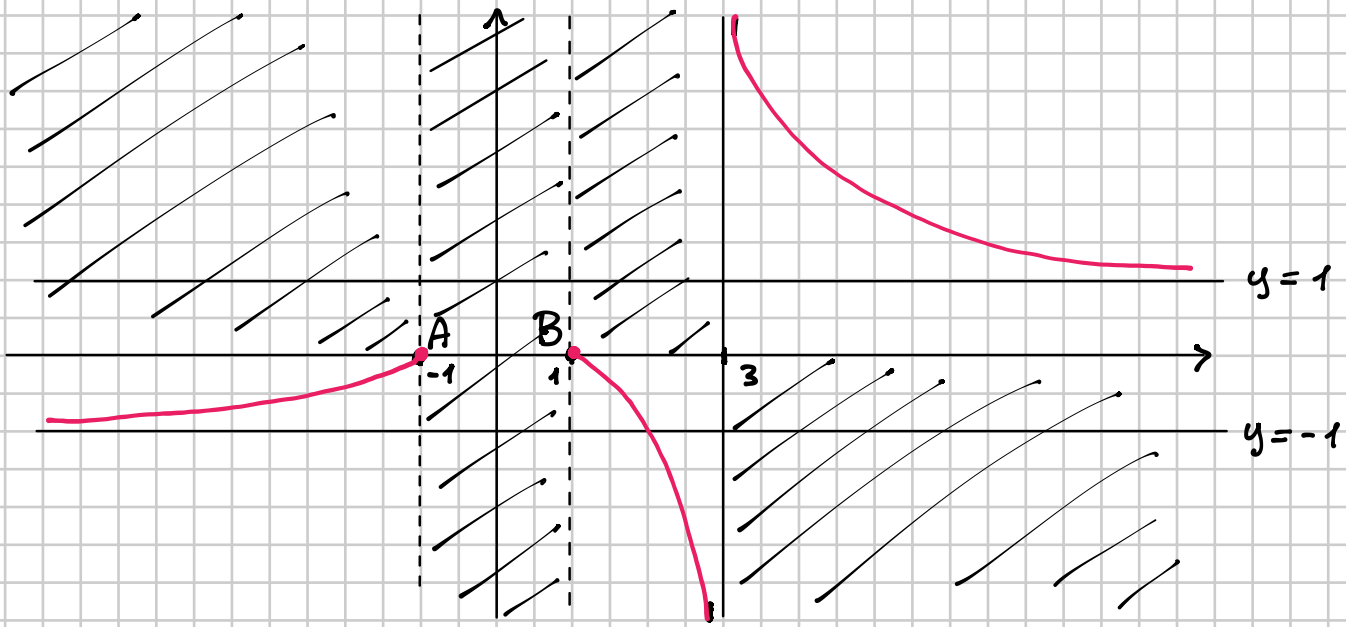


$\lim_{x \to -\infty} \arctan(x) = -\frac{\pi}{2}$ $\lim_{x \to +\infty} \arctan(x) = \frac{\pi}{2}$

STUDIO DI FUNZIONE

1053 $y = \dfrac{\sqrt{x^2 - 1}}{x - 3}$ (NÉ PARI NÉ DISPARI)

1) DOMINIO: $\begin{cases} x^2 - 1 \geq 0 \\ x - 3 \neq 0 \end{cases}$ $\begin{cases} x \leq -1 \ \vee \ x \geq 1 \\ x \neq 3 \end{cases}$ $D = (-\infty, -1] \cup [1, 3) \cup (3, +\infty)$



2) INTERSEZ. CON ASSI

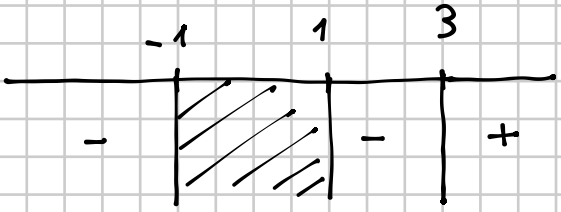
$\begin{cases} y = 0 \\ y = \dfrac{\sqrt{x^2-1}}{x-3} \end{cases}$ $\dfrac{\sqrt{x^2-1}}{x-3} = 0 \Rightarrow \sqrt{x^2-1} = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$

$A(-1, 0)$ $B(1, 0)$

3) SEGNO — la radice, dove esiste, è sempre ≥ 0, quindi il segno è determinato da $x-3$

$$\frac{\sqrt{x^2-1}}{x-3} > 0$$

$$\begin{cases} x < -1 \ \vee \ x > 1 \\ x - 3 > 0 \end{cases} \qquad \begin{cases} x < -1 \ \vee \ x > 1 \\ x > 3 \end{cases}$$



4) LIMITI

$$\lim_{x\to-\infty} \frac{\sqrt{x^2-1}}{x-3} = \lim_{x\to-\infty} \frac{\sqrt{x^2\left(1-\frac{1}{x^2}\right)}}{x\left(1-\frac{3}{x}\right)} = \lim_{x\to-\infty} \frac{|x|\sqrt{1-\frac{1}{x^2}}}{x\left(1-\frac{3}{x}\right)} =$$

$$= \lim_{x\to-\infty} \frac{-x\sqrt{1-\left(\frac{1}{x^2}\right)}}{x\left(1-\left(\frac{3}{x}\right)\right)} = -1 \qquad \left(\sqrt{1-\tfrac{1}{x^2}} \to 0,\ \tfrac{3}{x} \to 0\right)$$

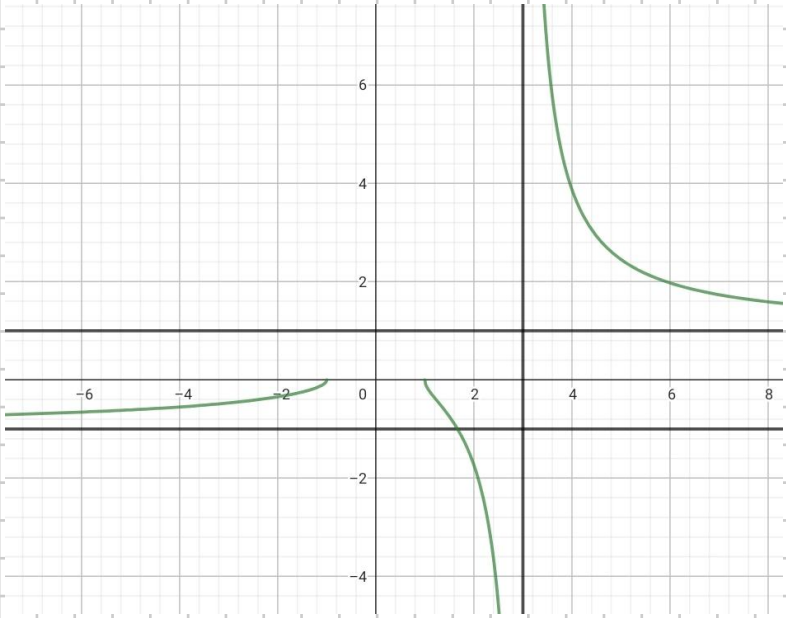
$y = -1$ è ASINTOTO ORIZZ. per $x \to -\infty$

$$\lim_{x\to+\infty} \frac{\sqrt{x^2-1}}{x-3} = \ldots = \lim_{x\to+\infty} \frac{|x|\sqrt{1-\frac{1}{x^2}}}{x\left(1-\frac{3}{x}\right)} = \lim_{x\to+\infty} \frac{x\sqrt{1-\left(\frac{1}{x^2}\right)}}{x\left(1-\left(\frac{3}{x}\right)\right)} = 1$$

$\left(\sqrt{1-\tfrac{1}{x^2}} \to 0,\ \tfrac{3}{x} \to 0\right)$

$y = 1$ è ASINTOTO ORIZZ. per $x \to +\infty$

$$\lim_{x\to3^-} \frac{\sqrt{x^2-1}}{x-3} = \frac{\sqrt{8}}{0^-} = -\infty \qquad \lim_{x\to3^+} \frac{\sqrt{x^2-1}}{x-3} = \frac{\sqrt{8}}{0^+} = +\infty$$



1067 $y = \ln \dfrac{x}{x+3}$

STUDIO DI FUNZ.

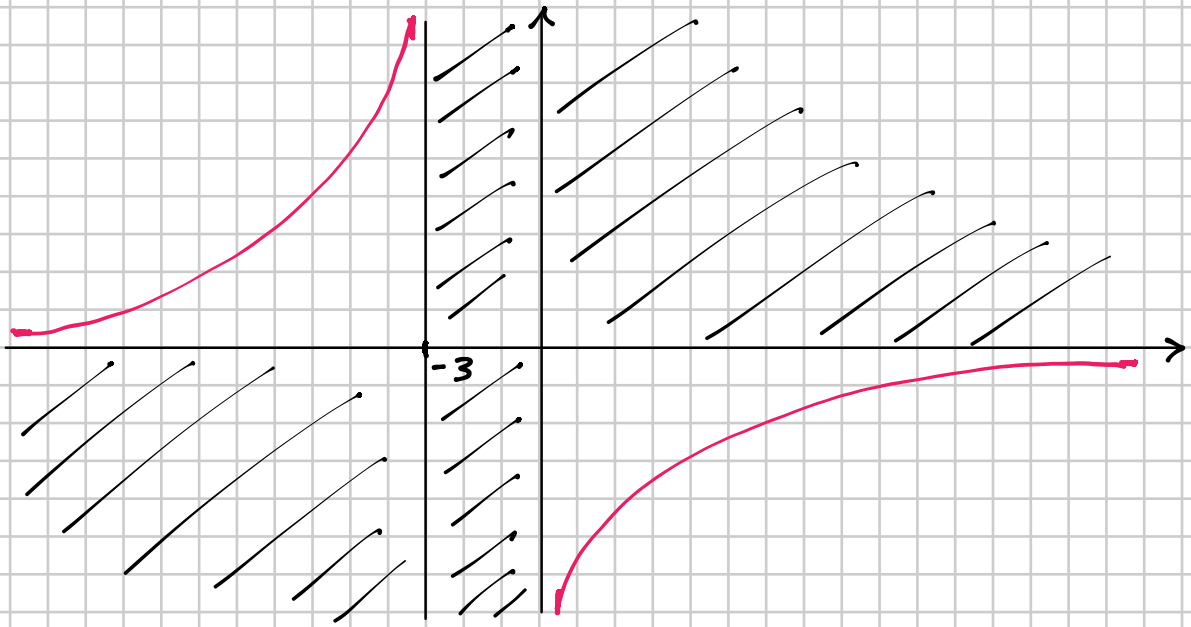
1) DOMINIO: $\dfrac{x}{x+3} > 0$

$N > 0 \quad x > 0$

$D > 0 \quad x+3 > 0 \Rightarrow x > -3$

	-3		0	
$-$	$\vert$	$-$	$\vert$	$+$
$-$	$\vert$	$+$	$\vert$	$+$
$+$	$\vert$	$-$	$\vert$	$+$

$D = (-\infty, -3) \cup (0, +\infty)$ $\qquad x < -3 \ \vee \ x > 0$



2) INT. ASSE X

$\begin{cases} y = 0 \\ y = \ln \dfrac{x}{x+3} \end{cases}$ $\qquad \ln \dfrac{x}{x+3} = 0 \qquad \begin{cases} \dfrac{x}{x+3} = 1 \\ x < -3 \ \vee \ x > 0 \end{cases}$

$\cancel{x} = \cancel{x} + 3 \qquad 0 = 3$ IMPOSS.

NON CI SONO INTERSEZIONI CON L'ASSE X

3) SEGNO

$\ln \dfrac{x}{x+3} > 0 \qquad \begin{cases} \dfrac{x}{x+3} > 1 \\ x < -3 \ \vee \ x > 0 \end{cases}$ $\qquad \begin{cases} \dfrac{x}{x+3} - 1 > 0 \\ \dots \end{cases}$ $\qquad \dfrac{\cancel{x} - \cancel{x} - 3}{x+3} > 0$

$\dfrac{3}{x+3} < 0$

$x + 3 < 0$

$x < -3$

$\begin{cases} x < -3 \\ x < -3 \ \vee \ x > 0 \end{cases} \Rightarrow x < -3$

	-3		0	
$+$	$\vert$	///	$\vert$	$-$

4) LIMITI

$\lim_{x\to-\infty} \ln\frac{x}{x+3} = \ln(1) = 0$ $\quad$ $y=0$ è ASINTOTO ORIZZ. per $x\to\pm\infty$

$\lim_{x\to+\infty} \ln\frac{x}{x+3} = \ln(1) = 0$

$\lim_{x\to-3^-} \ln\frac{x}{x+3} = \ln\left(\frac{-3}{0^-}\right) = \underbrace{\ln(+\infty)}_{\text{si intende } \lim_{x\to+\infty}\ln(x)} = +\infty$ $\quad$ $x=-3$ è ASINTOTO VERT.

$\lim_{x\to0^+} \ln\frac{x}{x+3} = \underbrace{\ln(0^+)}_{\text{significa } \lim_{x\to0^+}\ln x} = -\infty$ $\quad$ $x=0$ è ASINTOTO VERT.