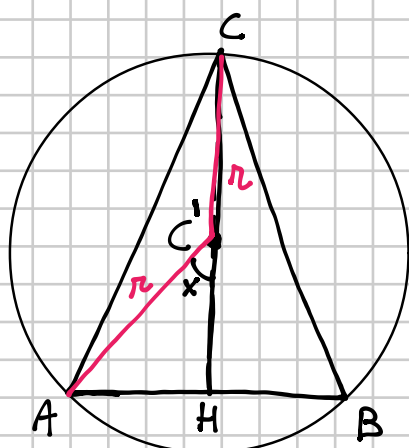


467 Fra tutti i triangoli isosceli inscritti in un cerchio di raggio r , trova quello di area massima.

[il triangolo equilatero con altezza che misura $\frac{3}{2}r$]



$$\overline{AB} = 2 \overline{AH} = 2r \sin x$$

$$0 \leq x \leq \pi$$

$$\overline{CH} = r + r \cos x$$

$$A(x) = \frac{1}{2} (2r \sin x) (r + r \cos x) = r^2 \sin x (1 + \cos x)$$

$$x \in [0, \pi]$$

$$A'(x) = r^2 [\cos x (1 + \cos x) + \sin x (-\sin x)] =$$

$$= r^2 [\cos x + \cos^2 x - \sin^2 x]$$

$$A'(x) = 0$$

in $[0, \pi]$

$$r^2 [\cos x + \cos^2 x - \sin^2 x] = 0$$

$$\cos x + 2 \cos^2 x - 1 = 0$$

$$2 \cos^2 x + \cos x - 1 = 0 \quad \Delta = 1 + 8 = 9$$

$$\cos x = \frac{-1 \pm 3}{4} = \begin{cases} -1 \Rightarrow x = \pi \\ \frac{1}{2} \Rightarrow x = \frac{\pi}{3} \end{cases}$$

$$A'(x) > 0$$

in $[0, \pi]$

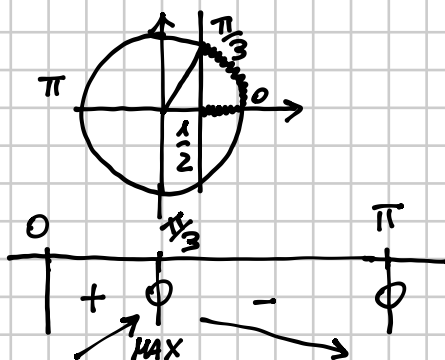
$$2 \cos^2 x + \cos x - 1 > 0$$

$$\cos x < -1 \quad \vee \quad \cos x > \frac{1}{2}$$

IMPOSS.

$$\Downarrow$$

$$0 < x < \frac{\pi}{3}$$

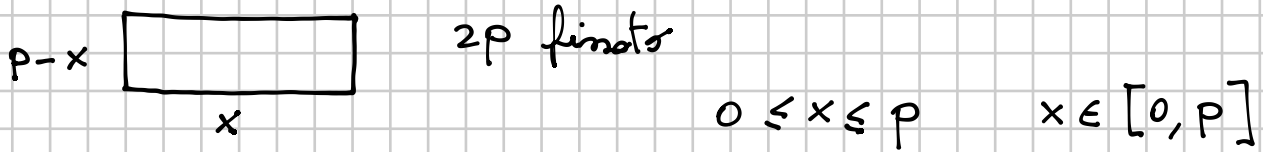


Il punto di MAX

si ha in $x = \frac{\pi}{3} \Rightarrow \overline{CH} = \frac{3}{2}r$

PROBLEMA CLASSICO

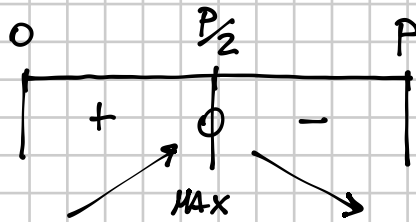
Es i tutti i rettangoli di perimetro fissato, qual è quello di area massima?



$$A(x) = (p-x)x = px - x^2 \quad \text{funzione d'area in } [0, p]$$

$$A'(x) = p - 2x \quad A'(x) = 0 \Rightarrow p - 2x = 0 \Rightarrow x = \frac{p}{2}$$

$$A'(x) > 0 \Rightarrow p - 2x > 0 \quad x < \frac{p}{2}$$



Il rettangolo di area massima si ha per $x = \frac{p}{2}$

$$\text{BASE} = \frac{p}{2}$$

↓
QUADRATO

$$\text{ALTEZZA} = p - \frac{p}{2} = \frac{p}{2}$$