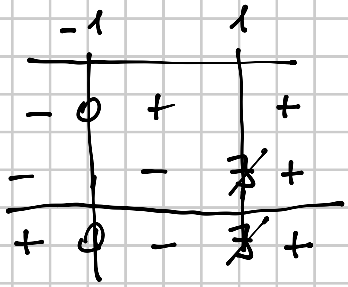
STUDIO DI FUNZIONE

111 $y = \sqrt{\dfrac{x+1}{x-1}}$

1) DOMINIO $\dfrac{x+1}{x-1} \geq 0$

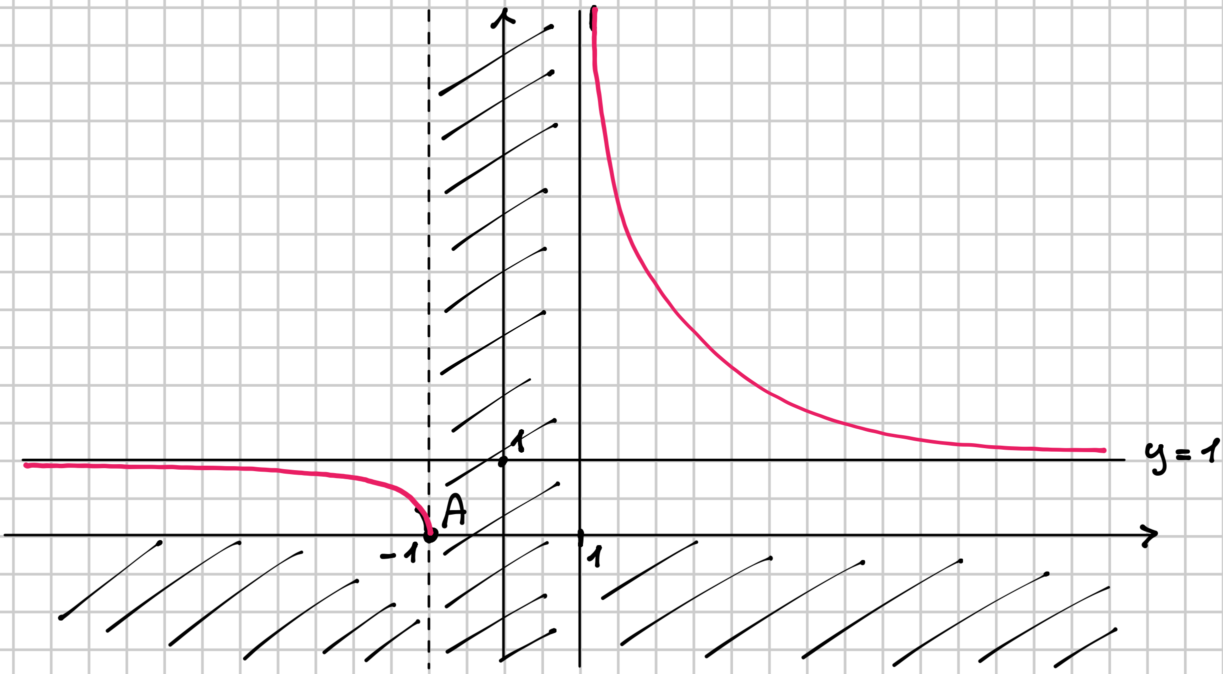
N] $x+1>0$ $\quad x>-1$

D] $x-1>0$ $\quad x>1$



$x \leq -1 \quad \vee \quad x>1$

$D = (-\infty, -1] \cup (1, +\infty)$



2) INT. ASSI

$$\begin{cases} y=0 \\ y=\sqrt{\dfrac{x+1}{x-1}} \end{cases} \Rightarrow \sqrt{\dfrac{x+1}{x-1}}=0 \Rightarrow x=-1 \qquad A(-1,0)$$

3) SEGNO

$\sqrt{\dfrac{x+1}{x-1}} \geq 0 \quad \forall x \in D \qquad f(x)>0 \quad \forall x \in D \setminus \{-1\}$

4) LIMITI

$\lim\limits_{x \to \pm\infty} \sqrt{\dfrac{x+1}{x-1}} = 1 \qquad y=1$ ASINTOTO ORIZZONTALE per $x \to \pm\infty$

$$\lim_{x\to -1^-} \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{0^-}{-2}} = 0^+$$ f è CONTINUA IN $-1 \Rightarrow (f(-1)=0)$

$$\lim_{x\to 1^+} \sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{2}{0^+}} = +\infty$$ $x = 1$ ASINTOTO VERTICALE

5) DERIVATA PRIMA

$$f(x) = \sqrt{\frac{x+1}{x-1}}$$

$$f'(x) = \frac{1}{2\sqrt{\frac{x+1}{x-1}}} \cdot \left(\frac{x+1}{x-1}\right)' = \frac{1}{2}\sqrt{\frac{x-1}{x+1}} \cdot \frac{x-1-(x+1)}{(x-1)^2} =$$

$x \in (-\infty,-1) \cup (1,+\infty)$

$$= \frac{1}{2}\sqrt{\frac{x-1}{x+1}} \cdot \frac{\cancel{x}-1-\cancel{x}-1}{(x-1)^2} =$$

$$= \frac{1}{\cancel{2}}\sqrt{\frac{x-1}{x+1}} \cdot \frac{-\cancel{2}}{(x-1)^2} =$$

$$= -\frac{1}{(x-1)^2}\sqrt{\frac{x-1}{x+1}}$$

NON CI SONO ZERI DI f'

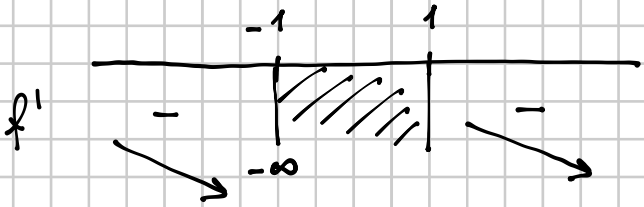
$f'(x) < 0 \quad \forall x \in (-\infty,-1) \cup (1,+\infty)$

Vediamo cosa succede in $x=-1$

$$\lim_{x\to -1^-} f'(x) = \lim_{x\to -1^-} -\frac{1}{(x-1)^2}\sqrt{\frac{x-1}{x+1}} = -\frac{1}{4}\cdot\overbrace{\sqrt{\frac{-2}{0^-}}}^{+\infty} = -\infty$$

$x=-1$ punto di non derivabilità con tangente verticale

P.TO DI MINIMO



P.TO DI MINIMO

6) DERIVATA SECONDA

$$f'(x) = -\frac{1}{(x-1)^2}\sqrt{\frac{x-1}{x+1}} = -(x-1)^{-2}\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$$

$$f''(x) = 2(x-1)^{-3}\cdot\sqrt{\frac{x-1}{x+1}} - \frac{1}{(x-1)^2}\cdot\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-\frac{1}{2}}\cdot\left(\frac{x-1}{x+1}\right)' =$$

$$= \frac{2}{(x-1)^3}\sqrt{\frac{x-1}{x+1}} - \frac{1}{2(x-1)^2}\sqrt{\frac{x+1}{x-1}}\cdot\frac{\cancel{x}+1-\cancel{x}+1}{(x+1)^2} =$$

$$= \frac{2}{(x-1)^3}\sqrt{\frac{x-1}{x+1}} - \frac{1}{(x-1)^2(x+1)^2}\sqrt{\frac{x+1}{x-1}} =$$

$$= \frac{1}{(x-1)^2}\sqrt{\frac{x-1}{x+1}}\left[\frac{2}{x-1} - \frac{1}{(x+1)^2}\frac{\sqrt{\frac{x+1}{x-1}}}{\sqrt{\frac{x-1}{x+1}}}\right] =$$

$$= \frac{1}{(x-1)^2}\sqrt{\frac{x-1}{x+1}}\left[\frac{2}{x-1} - \frac{1}{(x+1)^2}\sqrt{\frac{x+1}{x-1}\cdot\frac{x+1}{x-1}}\right] =$$

$$= \frac{1}{(x-1)^2}\sqrt{\frac{x-1}{x+1}}\left[\frac{2}{x-1} - \frac{1}{(x+1)^2}\sqrt{\left(\frac{x+1}{x-1}\right)^2}\right] =$$

$\left|\frac{x+1}{x-1}\right| = \frac{x+1}{x-1}$ perché in D $\frac{x+1}{x-1} \geq 0$

$$= \frac{1}{(x-1)^2}\sqrt{\frac{x-1}{x+1}}\left[\frac{2}{x-1} - \frac{1}{(x+1)^{\cancel{2}}}\cdot\frac{\cancel{x+1}}{x-1}\right] =$$

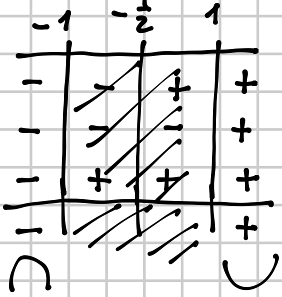
$$= \frac{1}{(x-1)^2}\sqrt{\frac{x-1}{x+1}}\;\frac{2x+2-1}{(x-1)(x+1)} = \underbrace{\frac{1}{(x-1)^2}\sqrt{\frac{x-1}{x+1}}}_{>0}\;\frac{2x+1}{(x-1)(x+1)}$$

$\frac{2x+1}{(x-1)(x+1)} > 0$

$2x+1>0 \quad x>-\frac{1}{2}$

$x-1>0 \quad x>1$

$x+1>0 \quad x>-1$



NON CI SONO PUNTI DI FLESSO