181 $\int \frac{\sin x}{\cos x + 2} dx$ $[-\ln(\cos x + 2) + c]$

È del tipo $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$

$$\int \frac{\sin x}{\cos x + 2} dx = - \int \frac{-\sin x}{\cos x + 2} dx = -\ln|\cos x + 2| + c =$$

$$= -\ln(\cos x + 2) + c$$

Calcoliamo:

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = - \int \frac{-\sin x}{\cos x} dx = -\ln|\cos x| + c$$

(in modo simile)

275 $\int \frac{9x - 3}{x^2 + 1} dx =$ $\left[\frac{9}{2}\ln(x^2 + 1) - 3\arctan x + c\right]$

$$= \int \frac{9x}{x^2+1} dx - 3 \int \frac{1}{x^2+1} dx =$$

$$= \frac{9}{2} \int \frac{2x}{x^2+1} dx - 3 \int \frac{1}{x^2+1} dx = \frac{9}{2} \ln|x^2+1| - 3 \arctan x + c =$$

$$= \frac{9}{2} \ln(x^2+1) - 3 \arctan x + c$$

308 $\displaystyle\int \frac{e^x \tan e^x}{\cos^2 e^x}\,dx = \int (\tan e^x)' \cdot \tan e^x \, dx = (*)$

$$(\tan e^x)' = \frac{1}{\cos^2 e^x} \cdot e^x$$

$$\boxed{\int f'(x) \cdot f(x)\, dx = \frac{1}{2}[f(x)]^2 + c}$$

$$(*) = \frac{1}{2}[\tan e^x]^2 + c = \frac{1}{2}\tan^2 e^x + c$$

309 $\displaystyle\int \frac{x}{1+4x^4}\,dx =$

$$= \int \frac{x}{1+(2x^2)^2}\,dx = \frac{1}{4}\int \frac{4x}{1+(2x^2)^2}\,dx = \frac{1}{4}\arctan(2x^2) + c$$

292 $\displaystyle\int \frac{4x+x^3}{\sqrt{1-x^4}}\,dx =$

$$= \int \frac{4x+x^3}{\sqrt{1-(x^2)^2}}\,dx = \int \frac{4x}{\sqrt{1-(x^2)^2}}\,dx + \int \frac{x^3}{\sqrt{1-x^4}}\,dx =$$

$$= 2\int \frac{2x}{\sqrt{1-(x^2)^2}}\,dx + \frac{1}{4}\int \frac{4x^3}{\sqrt{1-x^4}}\,dx =$$

$$= 2\arcsin x^2 + \frac{1}{2}\int \frac{4x^3}{2\sqrt{1-x^4}}\,dx = 2\arcsin x^2 - \frac{1}{2}\int \frac{-4x^3}{2\sqrt{1-x^4}}\,dx =$$

$$= 2\arcsin x^2 - \frac{1}{2}\sqrt{1-x^4} + c$$

INTEGRAZIONE PER SOSTITUZIONE

348 $\int \frac{1+e^{\sqrt{x}}}{\sqrt{x}}dx$; $t=\sqrt{x}$.

$t=\sqrt{x}$

$x=t^2$

$\frac{dx}{dt}=2t$

$dx = 2t\,dt$

$$\int \frac{1+e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int \frac{1+e^{t}}{\cancel{t}}\,2\cancel{t}\,dt =$$

$$= 2\int (1+e^t)\,dt =$$

$$= 2\left[t+e^t\right]+c = \boxed{2\left(\sqrt{x}+e^{\sqrt{x}}\right)+c}$$

372 $\int \sqrt{1+2\cos x}\,\sin x\,dx$; $t=\cos x$.

$\left[-\frac{1}{3}(1+2\cos x)^{\frac{3}{2}}+c\right]$

$t=\cos x$

$x=\arccos t$

$\frac{dx}{dt} = -\frac{1}{\sqrt{1-t^2}}$

$dx = -\frac{1}{\sqrt{1-t^2}}\,dt$

$\sin x = \sin(\arccos t) =$

$= \sqrt{1-\cos^2(\arccos t)} = \sqrt{1-t^2}$

$$= \int \sqrt{1+2t}\cdot\cancel{\sqrt{1-t^2}}\left(-\frac{1}{\cancel{\sqrt{1-t^2}}}\right)dt$$

$$= -\int \sqrt{1+2t}\,dt =$$

$$= -\int (1+2t)^{\frac{1}{2}}\,dt =$$

$$= -\frac{1}{2}\int 2(1+2t)^{\frac{1}{2}}\,dt =$$

$$= -\frac{1}{2}\,\frac{1}{\frac{1}{2}+1}\,(1+2t)^{\frac{1}{2}+1}+c =$$

$$= -\frac{1}{\cancel{2}}\,\frac{1}{\frac{3}{\cancel{2}}}\,(1+2t)^{\frac{3}{2}}+c = -\frac{1}{3}\sqrt{(1+2t)^3}+c = \boxed{-\frac{1}{3}\sqrt{(1+2\cos x)^3}+c}$$