383 $\int \frac{2e^{2x}}{1+e^{x}}dx =$ $[2e^x - 2\ln(e^x+1)+c]$

$$= \int \frac{2t^{\cancel{2}}}{1+t}\cdot\frac{1}{\cancel{t}}\,dt = 2\int\frac{t}{1+t}\,dt = 2\int\frac{t+1-1}{t+1}\,dt =$$

$t = e^x \qquad x = \ln t$

$\frac{dx}{dt} = \frac{1}{t} \qquad dx = \frac{1}{t}\,dt$

$$= 2\int\left(\frac{t+1}{t+1} - \frac{1}{t+1}\right)dt =$$

$$= 2\int\left(1 - \frac{1}{t+1}\right)dt =$$

$$= 2\int dt - 2\int\frac{1}{t+1}\,dt =$$

$$= 2t - 2\ln|t+1| + c =$$

$$= 2e^x - 2\ln|e^x+1| + c =$$

$$= \boxed{2e^x - 2\ln(e^x+1) + c}$$

387 $\int \frac{4}{1+\cos x}dx = (*)$ $\left[4\tan\frac{x}{2}+c\right]$

FORMULE PARAMETRICHE

$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2}$

$t = \tan\frac{x}{2}$

$\frac{x}{2} = \arctan t \qquad x = 2\arctan t$

$\frac{dx}{dt} = \frac{2}{1+t^2}$

$dx = \frac{2}{1+t^2}\,dt$

$$(*) = \int \frac{4}{1+\frac{1-t^2}{1+t^2}}\cdot\frac{2}{1+t^2}\,dt =$$

$$= \int \frac{4}{\frac{1+\cancel{t^2}+1-\cancel{t^2}}{\cancel{1+t^2}}}\cdot\frac{2}{\cancel{1+t^2}}\,dt =$$

$$= \int\frac{4}{\cancel{2}}\cdot\cancel{2}\,dt = \int 4\,dt = 4t + c =$$

$$= 4\tan\frac{x}{2} + c$$

399 $\int \sqrt{1-4x^2}\,dx =$

$$4x^2 = \sin^2 t$$
$$\Downarrow$$
$$2x = \sin t \quad (t = \arcsin 2x)$$
$$x = \frac{1}{2}\sin t$$
$$dx = \frac{1}{2}\cos t\,dt$$

$$= \int \sqrt{1-\sin^2 t}\cdot\frac{1}{2}\cos t\,dt =$$

$$= \int \cos t\cdot\frac{1}{2}\cos t\,dt =$$

$$= \frac{1}{2}\int \cos^2 t\,dt =$$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$
$$\Downarrow$$
$$2\cos^2\alpha = 1 + \cos 2\alpha$$
$$\Downarrow$$
$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$= \frac{1}{2}\int \frac{1+\cos 2t}{2}\,dt =$$

$$= \frac{1}{2}\left[\int \frac{1}{2}\,dt + \frac{1}{2}\int \cos 2t\,dt\right] =$$

$$= \frac{1}{2}\left[\frac{1}{2}t + \frac{1}{2}\int \left(\frac{1}{2}\sin 2t\right)'dt\right] =$$

$$= \frac{1}{4}t + \frac{1}{4}\left[\frac{1}{2}\sin 2t\right] + C =$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$= \frac{1}{4}\arcsin(2x) + \frac{1}{8}\sin\left(2\arcsin(2x)\right) + C =$$

$$= \frac{1}{4}\arcsin(2x) + \frac{1}{8}\cdot 2\sin\left(\arcsin(2x)\right)\cdot\cos\left(\arcsin(2x)\right) + C =$$

$$= \frac{1}{4}\arcsin(2x) + \frac{1}{4}\cdot 2x\sqrt{1-\sin^2(\arcsin(2x))} + C =$$

$$= \frac{1}{4}\arcsin(2x) + \frac{1}{2}x\sqrt{1-4x^2} + C$$