INTEGRAZIONE PER PARTI

$$[f(x)\cdot g(x)]' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$f'(x)\cdot g(x) = [f(x)\cdot g(x)]' - f(x)\cdot g'(x)$$

$$\int f'(x)\cdot g(x)\,dx = \int [f(x)\cdot g(x)]'\,dx - \int f(x)\cdot g'(x)\,dx$$

$$\boxed{\int f'(x)\cdot g(x)\,dx = f(x)\cdot g(x) - \int f(x)\cdot g'(x)\,dx}$$

FORMULA DI INTEGRAZIONE PER PARTI

ESEMPI

1) $$\int x\cdot e^x\,dx = \int \underbrace{(e^x)'}_{f'(x)}\cdot \underbrace{x}_{g(x)}\,dx = e^x\cdot x - \int e^x\cdot \underbrace{(x)'}_{1}\,dx =$$

$$= xe^x - \int e^x\,dx = xe^x - e^x + c$$

2) $$\int \ln x\,dx = \int \underset{(x)'}{1}\cdot \ln x\,dx = x\cdot \ln x - \int x\cdot(\ln x)'\,dx =$$

$$= x\ln x - \int x\cdot\frac{1}{x}\,dx =$$

$$= x\ln x - \int dx = x\ln x - x + c$$

408 $\int 3x\cos x\,dx =$ $[3x\sin x + 3\cos x + c]$

$$= 3\int x\cos x\,dx = 3\int x(\sin x)'\,dx = 3\left[x\sin x - \int 1\cdot\sin x\,dx\right] =$$

$$= 3\left[x\sin x - (-\cos x + c)\right] = 3\left[x\sin x + \cos x + c\right] =$$

$$= 3x\sin x + 3\cos x + c$$

436 $\int \frac{\ln x^2}{x^2}\,dx =$ $\left[-\frac{1}{x}(2+\ln x^2)+c\right]$

si può fare se $x>0$

$= \int \frac{1}{x^2}\, 2\ln x\, dx = 2\int \frac{1}{x^2}\cdot \ln x\, dx = 2\int x^{-2}\cdot \ln x\, dx =$

$= 2\int \left(\frac{x^{-2+1}}{-2+1}\right)'\cdot \ln x\, dx = 2\int (-x^{-1})'\cdot \ln x\, dx =$

$= 2\left[-x^{-1}\cdot \ln x - \int (-x^{-1})\cdot \frac{1}{x}\, dx\right] =$

$= -\frac{2}{x}\ln x + 2\int \frac{1}{x^2}\, dx = -\frac{2}{x}\ln x - \frac{2}{x} + c = \boxed{-\frac{2}{x}(\ln x + 1) + c}$

437 $\int 2x\arctan x\, dx =$ $[(x^2+1)\arctan x - x + c]$

$= \int (x^2)'\arctan x\, dx = x^2\arctan x - \int x^2\cdot \frac{1}{1+x^2}\, dx =$

$= x^2\arctan x - \int \frac{x^2}{x^2+1}\, dx = x^2\arctan x - \int \frac{x^2+1-1}{x^2+1}\, dx =$

$= x^2\arctan x - \int \frac{x^2+1}{x^2+1}\, dx + \int \frac{1}{x^2+1}\, dx =$

$= x^2\arctan x - x + \arctan x + c = \boxed{(x^2+1)\arctan x - x + c}$

444 $\int e^x \cos x \, dx$ $\left[\frac{e^x}{2}(\sin x + \cos x) + c\right]$

$$\underbrace{\int e^x}_{(e^x)'} \cos x \, dx = e^x \cos x - \int e^x \cdot (-\sin x) \, dx =$$

applico ancora la formula per parti

$$= e^x \cos x + \int e^x \sin x \, dx =$$

$$= e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

Sono arrivato a scrivere

$$\int e^x \cos x \, dx = e^x (\cos x + \sin x) - \int e^x \cos x \, dx$$

sposto cambiando segno

$$2 \int e^x \cos x \, dx = e^x (\cos x + \sin x) + c$$

DIVIDO PER 2

$$\int e^x \cos x \, dx = \frac{e^x}{2} (\cos x + \sin x) + c$$

451 Calcola $\int \cos^2 x \, dx$ in due modi: con l'integrazione per parti e con la sostituzione $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$.

$\left[\frac{x+\sin x\cos x}{2}+c\right]$

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\int \cos^2 x \, dx = \int \frac{1+\cos 2x}{2} \, dx =$$

$$= \int \frac{1}{2} \, dx + \frac{1}{2}\int \cos 2x \, dx =$$

$$= \frac{1}{2}x + \frac{1}{4}\int \underbrace{2\cos 2x}_{(\sin 2x)'} \, dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C =$$

$$= \frac{1}{2}x + \frac{1}{\cancel{4}_2}\cdot \cancel{2}\sin x\cos x + C = \frac{x+\sin x\cos x}{2} + C$$

Usando l'integrazione per parti:

$$\int \cos^2 x \, dx = \int \cos x \cdot \cos x \, dx = \int (\sin x)' \cdot \cos x \, dx =$$

$$= \sin x \cdot \cos x - \int \sin x \cdot (-\sin x) \, dx =$$

$$= \sin x \cos x + \int \sin^2 x \, dx =$$

$$= \sin x \cos x + \int (1-\cos^2 x) \, dx =$$

$$= \sin x \cos x + \int dx - \int \cos^2 x \, dx$$

$$2\int \cos^2 x \, dx = \sin x \cos x + x + C$$

$$\int \cos^2 x \, dx = \frac{x+\sin x\cos x}{2} + C$$