INTEGRAZIONE DI FUNZIONI RAZIONALI FRATTE

464 $\displaystyle\int \frac{3x+3}{x^2+2x+9}\,dx =$ $\left[\frac{3}{2}\ln(x^2+2x+9)+c\right]$

$\Delta < 0$

$$= \frac{3}{2}\int \frac{2(x+1)}{x^2+2x+9}\,dx = \frac{3}{2}\ln|x^2+2x+9| + c = \frac{3}{2}\ln(x^2+2x+9)+c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

478 $\displaystyle\int \frac{x^2+x+1}{x-4}\,dx = (*)$

$$\begin{array}{rrr|l} x^2 & +x & +1 & x-4 \\ \hline -x^2 & +4x & & x+5 \\ \hline // & 5x & +1 & \\ & -5x & +20 & \\ \hline & // & 21 & \end{array}$$

$$x^2+x+1 = (x+5)(x-4)+21$$

$$\frac{x^2+x+1}{x-4} = x+5+\frac{21}{x-4}$$

$$(*) = \int (x+5)\,dx + \int \frac{21}{x-4}\,dx = \boxed{\frac{1}{2}x^2+5x+21\ln|x-4|+c}$$

485 $\int \frac{3x-5}{x^2-2x-3}\,dx \quad [2\ln|x+1|+\ln|x-3|+c]$

$(x+1)(x-3)$

$$\frac{3x-5}{x^2-2x-3} = \frac{3x-5}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3} = \frac{A(x-3)+B(x+1)}{(x+1)(x-3)} =$$

$$= \frac{Ax-3A+Bx+B}{(x+1)(x-3)} = \frac{x(A+B)-3A+B}{(x+1)(x-3)} \qquad \begin{cases} A+B=3 \\ -3A+B=-5 \end{cases}$$

$$\begin{cases} A=3-B \\ -3(3-B)+B=-5 \end{cases} \quad \begin{cases} // \\ -9+3B+B=-5 \end{cases} \quad \begin{cases} // \\ 4B=4 \end{cases} \quad \begin{cases} A=2 \\ B=1 \end{cases}$$

$$\int \frac{3x-5}{x^2-2x-3}\,dx = \int \frac{2}{x+1}\,dx + \int \frac{1}{x-3}\,dx = \boxed{2\ln|x+1| + \ln|x-3| + c}$$

494 $\int \frac{2x-1}{x^2+2x+1}\,dx = \quad \left[\ln(x+1)^2 + \frac{3}{x+1} + c\right]$

$\Delta = 0$

$$= \int \frac{2x-1}{(x+1)^2}\,dx = \int \frac{2(t-1)-1}{t^2}\,dt = \int \frac{2t-2-1}{t^2}\,dt =$$

SOSTITUZIONE

$t = x+1$

$x = t-1$

$dx = dt$

$$= \int \frac{2t-3}{t^2}\,dt = \int \frac{2t}{t^2}\,dt - \int \frac{3}{t^2}\,dt =$$

$$= 2\int \frac{1}{t}\,dt - 3\int t^{-2}\,dt =$$

$$= 2\ln|t| - 3\cdot\frac{1}{-2+1}\,t^{-2+1} + c = 2\ln|t| + \frac{3}{t} + c = \boxed{2\ln|x+1| + \frac{3}{x+1} + c}$$

492 $\int \frac{3}{x^2 - 10x + 25}\,dx =$ $\left[\frac{3}{5-x} + c\right]$

$$= \int \frac{3}{(x-5)^2}\,dx = \int \frac{3}{t^2}\,dt = 3\int t^{-2}\,dt =$$

$t = x - 5$

$x = t + 5$

$dx = dt$

$$= 3 \cdot \frac{1}{-2+1}\, t^{-2+1} + c =$$

$$= -\frac{3}{t} + c = -\frac{3}{x-5} + c = \boxed{\frac{3}{5-x} + c}$$