486 $\int \frac{3x-9}{x^2-x-2}\,dx$ $\left[\ln\frac{|x+1|^4}{|x-2|}+c\right]$

$(x-2)(x+1)$

$$\frac{3x-9}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1} = \frac{A(x+1)+B(x-2)}{(x-2)(x+1)} = \frac{Ax+A+Bx-2B}{(x-2)(x+1)} =$$

$$= \frac{(A+B)x+A-2B}{(x-2)(x+1)} \Rightarrow \begin{cases} A+B=3 \\ A-2B=-9 \end{cases} \quad \begin{cases} A=3-B \\ 3-B-2B=-9 \end{cases}$$

$$\begin{cases} // \\ -3B=-12 \end{cases} \quad \begin{cases} A=-1 \\ B=4 \end{cases}$$

$$\int \frac{3x-9}{x^2-x-2}\,dx = -\int \frac{1}{x-2}\,dx + 4\int \frac{1}{x+1}\,dx =$$

$$= -\ln|x-2| + 4\ln|x+1| + c =$$

$$= -\ln|x-2| + \ln|x+1|^4 + c =$$

$$= \boxed{\ln\frac{|x+1|^4}{|x-2|} + c}$$

RICORDARE CHE

$$\int \frac{1}{1+x^2}\,dx = \arctan x + c$$

497 $\displaystyle\int \frac{1}{x^2+4x+5}\,dx = \int \frac{1}{x^2+4x+4+1}\,dx = \int \frac{1}{(x+2)^2+1}\,dx$

$\Delta = 16-20=-4<0$

NON È SCOMPONIBILE

$$= \boxed{\arctan(x+2) + c}$$

501 $\displaystyle\int \frac{4}{x^2+6x+11}\,dx =$ $\left[2\sqrt{2}\arctan\frac{x+3}{\sqrt{2}}+c\right]$

$\Delta = 36-44<0$

$$= \int \frac{4}{x^2+6x+9-9+11}\,dx = \int \frac{4}{(x+3)^2+2}\,dx =$$

$$= 4\int \frac{1}{2\left[\frac{(x+3)^2}{2}+1\right]}\,dx = \frac{4}{2}\int \frac{1}{\left(\frac{x+3}{\sqrt{2}}\right)^2+1}\,dx =$$

$$= 2\int \frac{1}{\left(\frac{x+3}{\sqrt{2}}\right)^2+1}\,dx = 2\int \frac{1}{t^2+1}\sqrt{2}\,dt = 2\sqrt{2}\int \frac{1}{t^2+1}\,dt =$$

$$= 2\sqrt{2}\arctan t + c =$$

$t = \frac{x+3}{\sqrt{2}}$ $\qquad \sqrt{2}\,t = x+3$

$x = \sqrt{2}\,t - 3$

$dx = \sqrt{2}\,dt$

$$= \boxed{2\sqrt{2}\arctan\left(\frac{x+3}{\sqrt{2}}\right) + c}$$

502 $\displaystyle\int \frac{-1}{4x^2+4x+5}\,dx = \quad \left[-\frac{1}{4}\arctan\left(x+\frac{1}{2}\right)+c\right]$

$\Delta = 16 - 80 < 0$

$$= -\int \frac{1}{4x^2+4x+1+4}\,dx = -\int \frac{1}{(2x+1)^2+4}\,dx =$$

$$= -\int \frac{1}{4\left[\frac{(2x+1)^2}{4}+1\right]}\,dx = -\frac{1}{4}\int \frac{1}{\left(\frac{2x+1}{2}\right)^2+1}\,dx =$$

$$= -\frac{1}{4}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+1}\,dx = \boxed{-\frac{1}{4}\arctan\left(x+\frac{1}{2}\right)+c}$$

OSSERVAZIONE

Se fosse stato

$$\int \frac{1}{4x^2+4x+7}\,dx = \int \frac{1}{4x^2+4x+1+6}\,dx = \int \frac{1}{(2x+1)^2+6}\,dx =$$

$$= \int \frac{1}{6\left[\frac{(2x+1)^2}{6}+1\right]}\,dx = \frac{1}{6}\int \frac{1}{\left(\frac{2x+1}{\sqrt{6}}\right)^2+1}\,dx = \frac{1}{6}\int \frac{1}{t^2+1}\cdot\frac{\sqrt{6}}{2}\,dt$$

$$= \frac{\sqrt{6}}{12}\arctan t + c =$$

$$t = \frac{2x+1}{\sqrt{6}}$$

$$= \boxed{\frac{\sqrt{6}}{12}\arctan\left(\frac{2x+1}{\sqrt{6}}\right)+c}$$

$$t\sqrt{6} = 2x+1$$

$$2x = \sqrt{6}\,t - 1$$

$$x = \frac{\sqrt{6}}{2}t - \frac{1}{2} \Rightarrow dx = \frac{\sqrt{6}}{2}\,dt$$

506 $\displaystyle\int \frac{2x+3}{x^2-6x+10}\,dx =$

$\Delta < 0$

$[\ln(x^2-6x+10) + 9\arctan(x-3) + c]$

derivata del denominatore

$$= \int \frac{2x-6+6+3}{x^2-6x+10}\,dx = \int \frac{2x-6}{x^2-6x+10}\,dx + 9\int \frac{1}{x^2-6x+9+1}\,dx$$

$$= \ln|x^2-6x+10| + 9\int \frac{1}{(x-3)^2+1}\,dx =$$

$\Delta < 0$ tolgo il modulo

$$= \ln(x^2-6x+10) + 9\arctan(x-3) + c$$

517 $\displaystyle\int \frac{x^2+5x+4}{x^3+3x^2+x-5}\,dx$

Scomponiamo il denominatore con Ruffini

$$\begin{array}{c|ccc|c} & 1 & 3 & 1 & -5 \\ 1 & & 1 & 4 & 5 \\ \hline & 1 & 4 & 5 & // \end{array}$$

$$(x^2+4x+5)(x-1)$$

$\Delta<0$ non ulteriormente scomponibile

$$\frac{x^2+5x+4}{(x-1)(x^2+4x+5)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4x+5} =$$

$$= \frac{A(x^2+4x+5)+(Bx+C)(x-1)}{(x-1)(x^2+4x+5)} = \frac{Ax^2+4Ax+5A+Bx^2-Bx+Cx-C}{(x-1)(x^2+4x+5)} =$$

$$= \frac{(A+B)x^2+(4A-B+C)x+5A-C}{(x-1)(x^2+4x+5)} \qquad \begin{cases} A+B=1 \\ 4A-B+C=5 \\ 5A-C=4 \end{cases}$$

$$\begin{cases} B=1-A \\ 4A-1+A+C=5 \\ 5A-C=4 \end{cases} \qquad \begin{cases} // \\ 5A+C=6 \\ 5A-C=4 \\ \hline 10A \quad =10 \end{cases} \qquad \begin{cases} A=1 \\ B=0 \\ C=1 \end{cases}$$

$$\int \frac{x^2+5x+4}{x^3+3x^2+x-5}\,dx = \int \frac{1}{x-1}\,dx + \int \frac{1}{x^2+4x+5}\,dx =$$

$$= \ln|x-1| + \int \frac{1}{(x+2)^2+1}\,dx = \boxed{\ln|x-1| + \arctan(x+2) + c}$$

TEST Dato il grafico di $y = f'(x)$, individua un possibile andamento del grafico della funzione $y = f(x)$.

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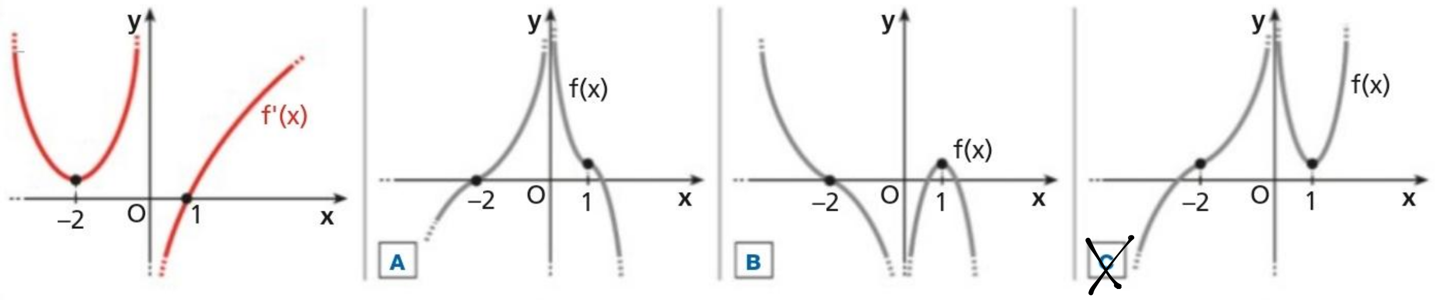

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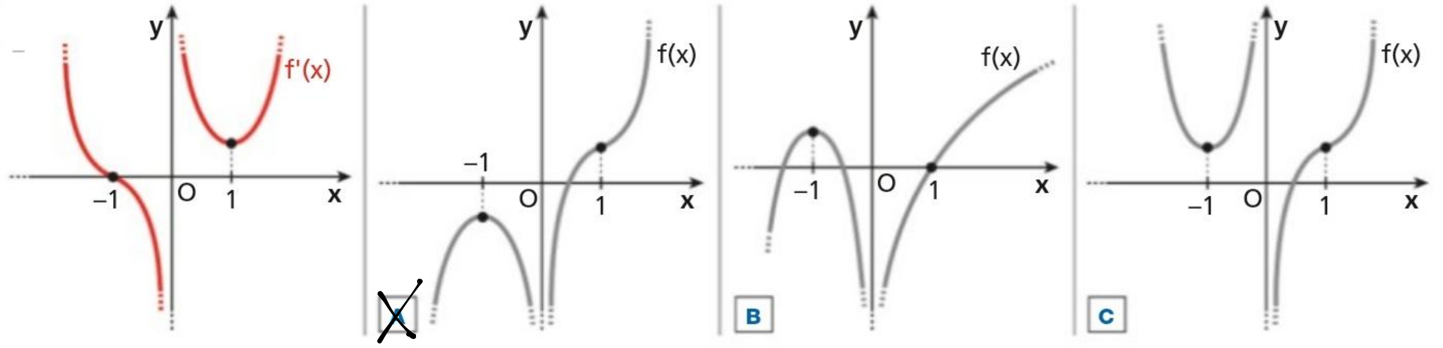

Dato il grafico della funzione $y = f'(x)$, traccia un possibile andamento della funzione $y = f(x)$ nei seguenti casi.

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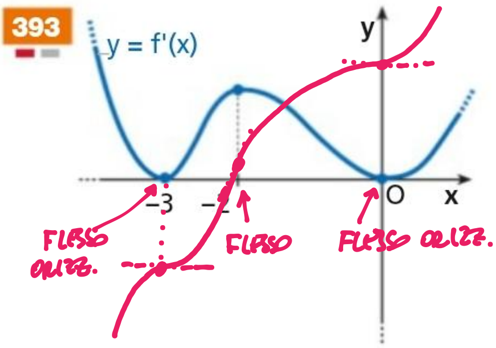

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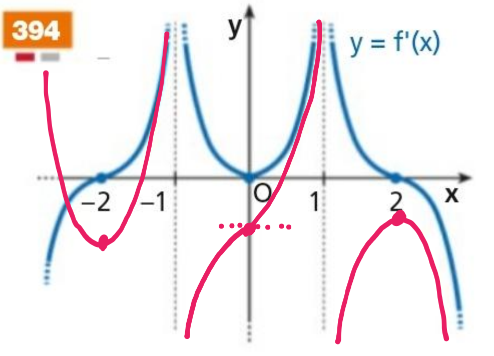

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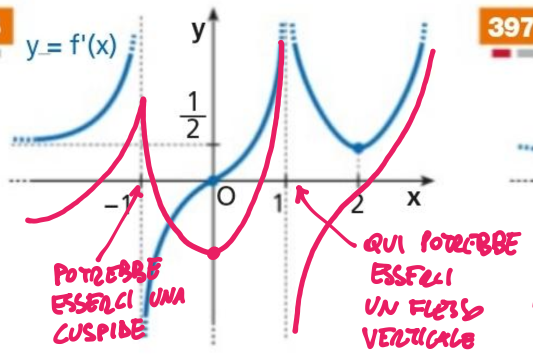

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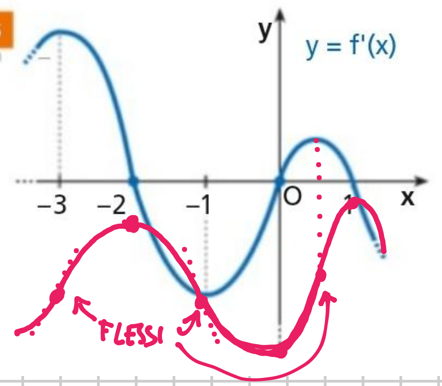

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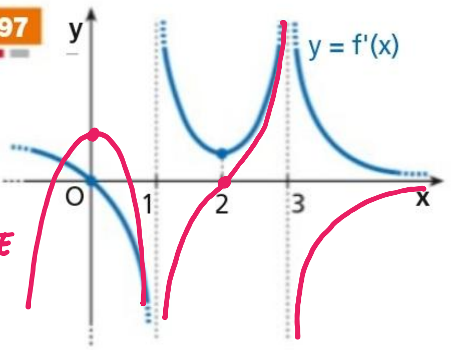

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