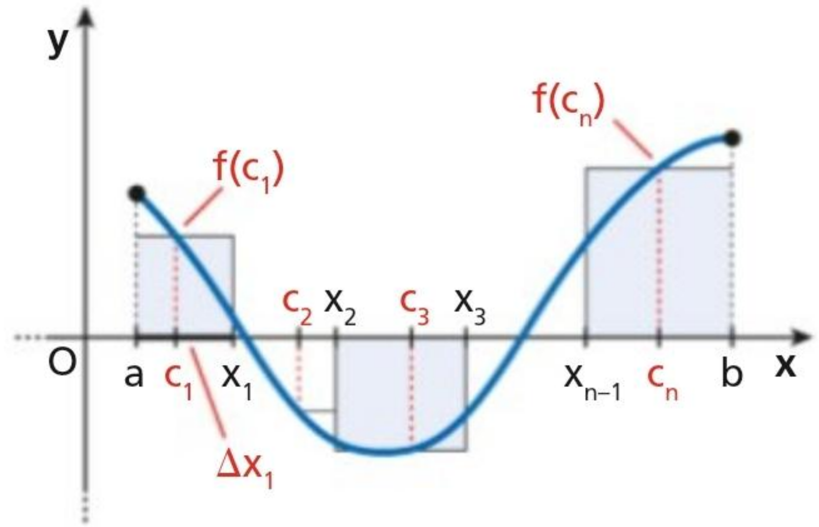
INTEGRALI DEFINITI



$f : [a, b] \to \mathbb{R}$

CONTINUA

Si suddivide l'intervallo $[a, b]$ in n parti

$a < x_1 < x_2 < x_3 < \ldots < x_{n-1} < b$

$[a, x_1], [x_1, x_2], \ldots, [x_{n-1}, b]$

n suddivisioni $(a = x_0$
$b = x_n)$

Si prende $c_k \in [x_{k-1}, x_k]$

Si dice SOMMA DI RIEMANN $\overline{S} = \sum_{k=1}^{n} f(c_k) \Delta x_k$ $\quad \Delta x_k = x_k - x_{k-1}$

$$\overline{S} = f(c_1) \cdot \Delta x_1 + f(c_2) \cdot \Delta x_2 + f(c_3) \cdot \Delta x_3 + \ldots + f(c_n) \cdot \Delta x_n.$$

DEFINIZIONE

Data una funzione $f(x)$, continua in $[a; b]$, l'**integrale definito** esteso all'intervallo $[a; b]$ è il valore del limite per Δx_{max} che tende a 0 della somma $\overline{S}$:

$$\int_a^b f(x)\,dx = \lim_{\Delta x_{max} \to 0} \overline{S}.$$

$\Delta x_{max} = \max \{\Delta x_1, \Delta x_2, \ldots, \Delta x_n\}$

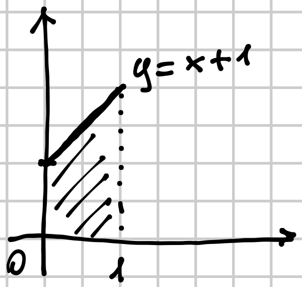
ANTICIPAZIONE

Come si calcolano gli integrali?

$$\boxed{\int_a^b f'(x)\,dx = f(b) - f(a)}$$

2° TEOREMA FONDAMENTALE DEL CALCOLO

1) $f: [0, 1]$ $\quad f(x) = x + 1$



applico la formula

$$\int_0^1 (x+1)\,dx = \int_0^1 \left(\tfrac{1}{2}x^2 + x\right)'dx = \tfrac{1}{2}x^2 + x \Big|_0^1 =$$

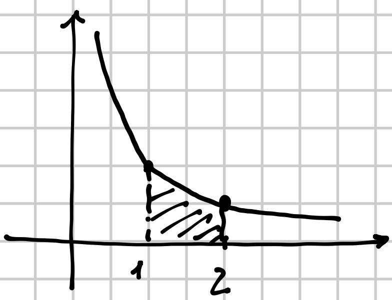
$$= \tfrac{1}{2}\cdot 1^2 + 1 - \left(\tfrac{1}{2}\cdot 0^2 + 0\right) =$$

$$= \tfrac{1}{2} + 1 = \tfrac{3}{2}$$

$$\int (x+1)\,dx = \underbrace{\tfrac{1}{2}x^2 + x + c}_{\text{PRIMITIVA}}$$

$$g(x)\Big|_a^b = g(b) - g(a)$$

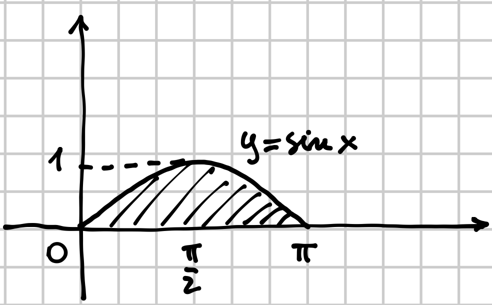
2) $f(x) = \frac{1}{x}$ $\quad f: [1, 2] \to \mathbb{R}$



$$\int_1^2 \frac{1}{x}\,dx = \int_1^2 (\ln x)'\,dx = \ln x \Big|_1^2 =$$

$$= \ln 2 - \underbrace{\ln 1}_{0} = \ln 2 \simeq 0{,}69315$$

3) $f(x) = \sin x$ $\quad f: [0, \pi] \to \mathbb{R}$

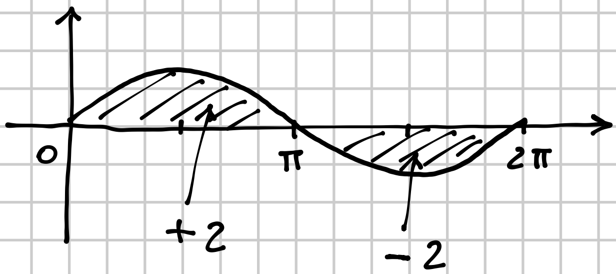


$$\int_0^\pi \sin x\,dx = \int_0^\pi (-\cos x)'\,dx =$$

$$= -\cos x \Big|_0^\pi = -\cos\pi - (-\cos 0) =$$

$$= 1 + 1 = 2$$

4) $g(x) = \sin x \qquad g: [0, 2\pi] \to \mathbb{R}$



$$\int_{\pi}^{2\pi} \sin x \, dx = -2$$

$$\int_0^{2\pi} \sin x \, dx = -\cos x \Big|_0^{2\pi} =$$

$$= -\cos 2\pi - (-\cos 0) =$$

$$= -1 + 1 = 0$$

VALGONO LE SEGUENTI PROPRIETÀ PER GLI INTEGRALI DEFINITI

$$\int_a^b (f(x) + g(x)) \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx$$

$$\int_a^b c f(x) \, dx = c \int_a^b f(x) \, dx$$

$$\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx \qquad a < c < b$$

$$\int_b^a f(x) \, dx = -\int_a^b f(x) \, dx \quad \text{(per definizione)}$$

$$f(x) \leq g(x) \implies \int_a^b f(x) \, dx \leq \int_a^b g(x) \, dx$$